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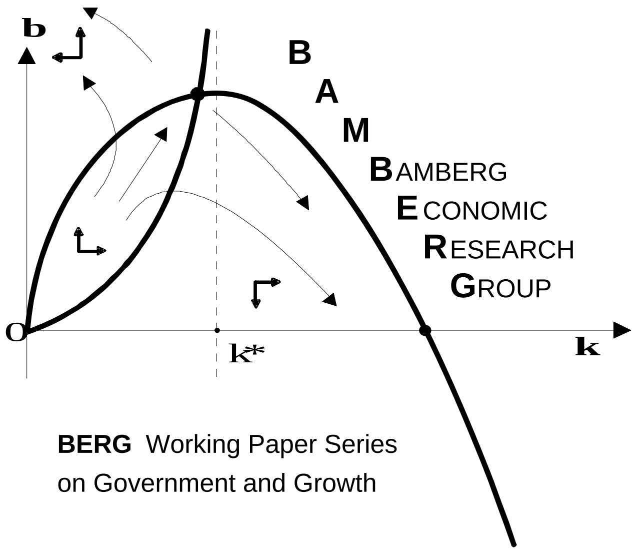

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A simple model of a speculative housing market*

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Abstract

We develop a simple model of a speculative housing market in which the demand for houses is influenced by expectations about future housing prices. Guided by empirical evidence, agents rely on extrapolative and regressive forecasting rules to form their expectations. The relative importance of these competing views evolves over time, subject to market circumstances. As it turns out, the dynamics of our model is driven by a two-dimensional nonlinear map which may display irregular boom and bust housing price cycles, as repeatedly observed in many actual markets. However, we also find that speculation may be a source of both stability and instability.

Keywords

Housing markets; Speculation; Boom and bust cycles; Nonlinear Dynamics.

JEL classification

D84; R21; R31.

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Introduction

As documented by Shiller (2005, 2007a, 2007b, 2008), although boom and bust home price cycles have occurred for centuries, the recent boom-bust development seems to dwarf anything seen before. Since the late 1990s, dramatic home price rallies have been observed in cities in countries such as Australia, Canada, China, France, India, Ireland, Italy, Korea, Russia, Spain, the United Kingdom, and the United States. Some of these price movements can be called spectacular. From 1996 to 2008, for instance, real home prices in London nearly tripled. Another impressive example concerns Las Vegas, where real home prices increased by 10 percent in 2003, followed by a 49 percent increase in 2004. For the United States as a whole, real home prices increased by 85 percent between 1997 and 2006. Then the United States' housing market burst and policy makers around the world are currently facing severe macroeconomic problems.

Shiller furthermore argues that this dramatic price increase is hard to explain from an economic point of view since economic fundamentals such as population growth, construction costs, interest rates or real rents do not match up with the observed home price increases. It is quite important to note that the boom of the early 2000s across cities and countries suggests that something very broad and general has been at work. This development cannot therefore be linked to factors specific to any of these markets. Shiller concludes that speculative thinking among investors, the use of heuristics such as extrapolative expectations, market psychology in the form of optimism and pessimism, herd behavior and social contagion of new ideas (new era thinking), and positive feedback dynamics are elements that play an important role in determining housing prices.

The goal of our paper is to develop a simple model of a speculative housing market to account for these observations. Our approach is inspired by recent work on agent-based financial market models (see Hommes 2006 and LeBaron 2006 for comprehensive surveys). In these models, the dynamics of financial markets depends on the expectation formation of boundedly rational heterogeneous interacting agents. As indicated by a number of empirical papers (summarized in Menkhoff and Taylor 2007), financial market participants rely on technical and fundamental trading rules when they determine their orders. Note that extrapolating technical trading rules add a positive feedback to the dynamics of financial markets and thus tend to be destabilizing. By predicting some kind of mean reversion, the effect of fundamental analysis is likely to be stabilizing. Within agent-based financial market models, the impact of these rules is usually time-varying – and it is precisely this that may give rise to complex endogenous dynamics.

For instance, in the models of Kirman (1991, 1993) and Lux (1995, 1997, 1998), agents switch between technical and fundamental analysis due to a herding mechanism, leading to periods where markets are relatively stable (dominance of fundamental analysis) or unstable (dominance of technical analysis). In Brock and Hommes (1997, 1998), the agents select their trading strategies with respect to their past profitability, i.e. this type of model incorporates an evolutionary learning process. Again, endogenous competition between trading strategies may lead to complex price dynamics. Other influential models include Day and Huang (1990), Chiarella (1992), de Grauwe et al. (1993), Chiarella et al. (2002), Westerhoff and Dieci (2006) and de Grauwe and Grimaldi (2006).

Such speculative forces are essential to our model. As pointed out by Shiller (2008), the same forces of human psychology that drive international financial markets also have the potential to affect other markets. In particular, this seems to be true for housing markets. Note that by now ample empirical evidence exists to show that human agents generally act in a boundedly rational manner (Kahneman et al. 1986, Smith 1991). Moreover, in many situations people seem to rely on rather simple heuristic principles when asked to forecast economic variables (Hommes et al. 2005, Heemeijer et al. 2008). The model we develop in this paper may thus be regarded as a stylized mathematical representation of what is going on in speculative housing markets. General theoretical and empirical evidence on (nonlinear) speculative bubbles is, for instance, provided by Rosser (1997, 2000).

The structure of our setup is as follows. We assume that housing prices adjust with respect to excess demand in the usual way. The supply of houses is determined by the depreciation of houses and new constructions, which, in turn, depend positively on housing prices. We discriminate between real and speculative demand for houses. As usual, real demand for houses depends negatively on housing prices. Speculative demand for houses is caused by agents' expected future housing prices. For simplicity, agents rely on only two heuristics when they make their predictions. Some agents believe that housing prices will return to a long-run fundamental steady state. However, other agents speculate on the persistence of bull and bear markets. The relative importance of these competing heuristics is due to market circumstances. To be precise, we assume that the more housing prices deviate from the long-run fundamental steady state, the more agents are convinced that some kind of mean reversion is about to set in. The underlying argument is that agents are aware that any bubble will ultimately burst, a situation where mean reversion rules predict the direction of the market movement correctly. A related rule selection scenario is used, for instance, in He and Westerhoff (2005) to understand the cyclical behavior of commodity prices.

The dynamics of our model is due to a two-dimensional nonlinear discrete-time dynamical system. We analytically show that our model may have up to three fixed points. Besides a so-called long-run fundamental steady state, two further steady states may also exist: one located below and one above this value. We are also able to determine the parameter space in which the long-run fundamental steady state is locally asymptotically stable. Interestingly, the impact of speculation on the stability of the housing market is ambiguous. There are parameter combinations where speculative forces stabilize an otherwise unstable fixed point (via a so-called subcritical flip bifurcation). However, for other possibly more realistic parameter combinations, the impact of speculation is destabilizing. The long-run fundamental steady state of our model may lose its stability via a so-called pitchfork bifurcation, after which two new nonfundamental steady states emerge, or via a so-called Neimark-Sacker bifurcation, after which (quasi-)periodic housing price dynamics set in. The latter scenario becomes more likely, the lower the rate of depreciation is. Finally, we present some numerical examples of boom and bust housing price cycles. These price paths appear to be quite irregular since both real and speculative forces jointly impact on the formation of housing prices and, in turn, realized prices affect agents' demand and supply decisions.

The paper is organized as follows. In section 2, we introduce a simple housing market model in which speculative forces are absent. In section 3, the model is extended and includes the expectation formation behavior of heterogeneous agents. Section 4 concludes our paper. A number of results are derived in the appendix.

2 The model without speculation

In this section, we first present our basic housing market model without speculative activity. We also characterize the dynamical system of our model which drives housing prices and the stock of houses, i.e. the model's two state variables.

2.1 Setup

Housing prices evolve with respect to demand and supply. Using a standard linear price adjustment function, housing price P in period $t+1$ is modeled as

$$P_{t+1} = P_t + a(D_t - S_t)\,, \tag{1}$$

where $a > 0$ is a price adjustment parameter and D and S stand for the total demand and total supply of houses, respectively. Obviously, housing prices increase if demand exceeds supply, and vice versa. Without loss of generality, we set the scaling parameter $a = 1$.

The total demand for houses consists of two components

$$D_t = D_t^R + D_t^S\,, \tag{2}$$

where D_t^R is the real demand for houses and D_t^S is the speculative demand for houses.

The real demand for houses is expressed as

$$D_t^R = b - cP_t\,. \tag{3}$$

Parameters b and c are both positive. As usual, demand depends negatively on the (current) price. In this section, we set $D_t^S = 0$, i.e. we exclude speculative forces for the moment.

The supply of houses is given as

$$S_t = S_{t-1} - (1-d)S_{t-1} + eP_t . \tag{4}$$

The second term on the right-hand side captures the depreciation of houses, where the rate of depreciation $(1-d)$ is limited to $0 < 1-d < 1$. The third term stands for the construction of new houses. Since $e > 0$, (4) states that the higher the price, the more new houses are built.

A few clarifying comments may be pertinent. Note that S and D are stock variables. The total supply of houses S thus also indicates the total stock of houses. Similarly, D represents the total demand for houses, or, put differently, the desired holding of houses. In the price adjustment equation (1), we match – in each time step – total demand and total supply quantities to determine the next period's housing price.

2.2 Dynamical system, fixed point and stability analysis

Recall that $D_t^S = 0$ and $a = 1$. Introducing the auxiliary variable $Z_{t+1} = S_t$, it is possible to reduce (1)-(4) to

$$\begin{cases} P_{t+1} = (1-c-e)P_t - dZ_t + b \\ Z_{t+1} = eP_t + dZ_t \end{cases}, \tag{5}$$

which is a two-dimensional discrete-time linear dynamical system.

Inserting $Z_{t+1} = Z_t = \overline{Z}$ and $P_{t+1} = P_t = \overline{P}$ into (5), we obtain the model's unique fixed point

$$\overline{Z} = \frac{e}{1-d}\overline{P} \tag{6}$$

and

$$\overline{P} = \frac{(1-d)b}{e+c(1-d)}. \tag{7}$$

It follows that $\overline{P}$ and $\overline{Z}$ are always positive. In the following, we call $\overline{P}$ the long-run fundamental steady state of our model, or simply the fundamental value. As revealed by (7), an increase in parameter b leads to an increase in the fundamental value, while an increase in parameters e, c and d yields the opposite, which is, of course, in agreement with common economic sense.

The parameter matrix of our linear map is given as

$$J = \begin{pmatrix} 1-c-e & -d \\ e & +d \end{pmatrix}, \tag{8}$$

where $tr = 1-c-e+d$ and $\det = d(1-c)$ denote the trace and the determinant of J, respectively. The fixed point of the linear model (5) is globally asymptotically stable if the following three conditions jointly hold (see, e.g. Medio and Lines 2001 and Gandolfo 2005): (i) $1+tr+\det>0$, (ii) $1-tr+\det>0$ and (iii) $1-\det>0$. Applying these conditions, we obtain

$$2-\frac{e}{1+d}-c>0, \tag{9}$$

$$c(1-d)+e>0, \tag{10}$$

and

$$1-d+cd>0. \tag{11}$$

Note that the latter two conditions are always true. Inequality (9) implies that the fixed point of our model may lose its stability when parameter c increases, parameter d decreases and parameter e increases. The stability domain of the fixed point is independent of parameter b. Again, this is consistent with economic intuition.

3 The model with speculation

Now we are ready to include speculative activity in our model. Afterwards, in subsection 3.2, we derive the model's dynamical system, its fixed points and the conditions for their local asymptotically stability. Section 3 ends with a few numerical examples of housing price bubbles and crashes.

3.1 Speculative demand

We assume that speculative forces entail an extrapolating and a mean reverting component. The relative importance of both components is time-varying since agents change their forecasting rules with respect to market circumstances. For simplicity, we do not track the activities of individual agents in this paper. Our approach may therefore also be interpreted as a model with a boundedly rational representative agent who uses a nonlinear mix of different forecasting rules. The representative agent then updates his/her mix in each time step. Note also that the total demand for houses in our model is simply given as the sum of the real demand for houses and the speculative demand for houses. For instance, if the speculative demand for houses is negative (positive), this decreases (increases) the total demand for houses. A negative speculative demand is not interpreted as short selling of houses in our model but as a correction term of the agents' real demand for houses. In our numerical examination we have verified that the total demand for houses is positive in any time step.

Speculative demand driven by the extrapolating component is formalized as

$$D_t^E = f(P_t - \overline{P}) . \tag{12}$$

The reaction parameter f is positive. When the housing price is above (below) its fundamental value, (12) implies that its followers optimistically (pessimistically) believe in a further price increase (decrease). Accordingly, their speculative demand is positive (negative). This simple yet elegant formulation goes back to Day and Huang (1990), and has been applied in a number of theoretical papers focussing on speculative dynamics. According to (12), rising prices lead to an increase in demand, i.e. the nature of (12) is indeed extrapolating.

Speculative demand generated by the mean-reverting component is written as

$$D_t^R = g(\overline{P} - P_t), \qquad (13)$$

where g is a positive reaction parameter. For instance, if the housing price is below its fundamental value, agents using (13) expect a price risee and consequently increase their demand for houses.

The total speculative demand is defined as

$$D_t^S = W_t D_t^E + (1 - W_t) D_t^R, \qquad (14)$$

where W and $1-W$ stand for the impacts of the extrapolation and mean reversion demand components. Recall that the total demand for houses (2) now consists of real demand for houses (3), buffeted by speculative demand for houses (14).

How do agents choose between the two speculative demand strategies? In this paper, they update their behavior in every time step with respect to market circumstances. The relative impact of extrapolators is formalized as

$$W_t = \frac{1}{1 + h(P_t - \overline{P})^2}, \qquad (15)$$

where h is a positive parameter. The intuition behind the bell-shaped curve (15) is as follows. Agents seek to exploit price trends (i.e. bull and bear markets). However, the more the price deviates from its fundamental value, the more agents come to the conclusion that a fundamental market correction is about to set in, and they consequently switch to the mean reverting predictor. Note that the higher parameter h, the faster the agents abandon extrapolating behavior as the mispricing increases (i.e. the tails of (15) decline with increasing h).

3.2 Dynamical system, fixed points and stability analysis

The results we now present are derived in the appendix. Let us define $\pi_t = P_t - \overline{P}$ and $\zeta_t = Z_t - \overline{Z}$. It is then possible to rewrite our model as a two-dimensional discrete-time nonlinear dynamical system

$$\begin{cases} \pi_{t+1} = (1-c-e)\pi_t + \dfrac{f\pi_t - gh\pi_t^3}{1+h\pi_t^2} - d\zeta_t \\ \zeta_{t+1} = e\pi_t + d\zeta_t \end{cases} . \tag{16}$$

Hence, in order to compute trajectories for π_t and ζ_t, an initial condition (π_0, ζ_0) has to be specified.

The dynamical system (16) may have up to three fixed points. For π we find

$$\overline{\pi}_1 = 0 \tag{17}$$

and

$$\overline{\pi}_{2,3} = \pm\sqrt{\frac{(1-d)(f-c)-e}{h(e+(1-d)(c+g))}} . \tag{18}$$

The denominator of (18) is always positive. The latter two fixed points thus only exist if $f > c + e/(1-d) > 0$ (implying a positive nominator). Hence, if the reaction parameter of the extrapolation rule exceeds a certain critical level, the model possesses three fixed points. The housing prices may then permanently be located above or below the fundamental steady state. For the model's second state variable, we obtain

$\bar{\zeta}_{1,2,3} = \frac{e}{1-d}\bar{\pi}_{1,2,3}$. Accordingly, the equilibrium supply of houses is relatively high (low) if the equilibrium housing price is located in the bull (bear) market. Should the price properly reflect its fundamental value, the supply of houses is as in section 3.1.

Moreover, it can be shown that the fixed point ($\bar{\pi}_1 = 0$, $\bar{\zeta}_1 = 0$) is locally asymptotically stable if the following inequalities jointly hold

$$f > c + \frac{e}{1+d} - 2, \tag{19}$$

$$f < c + \frac{e}{1-d}, \tag{20}$$

and

$$f < c + \frac{1}{d} - 1. \tag{21}$$

What is interesting here is that when the first inequality is violated, since f drops below a certain critical level (but the other two inequalities hold), we observe a (subcritical) flip bifurcation. When the second inequality is violated, since f increases (but the other two inequalities hold), we observe a (supercritical) pitchfork bifurcation. Finally, when the third inequality is violated, since f increases (but the other two inequalities hold), we observe a (supercritical) Neimark-Sacker bifurcation.

Let us illustrate this interesting finding. Figure 1 shows four bifurcation diagrams in which we vary the bifurcation parameter f as indicated on the axis. The other parameters are given in table 1. The first panel reveals that the fundamental steady state becomes (locally) attracting if f becomes larger than 0.1. Hence, speculative forces have a stabilizing impact in this situation.

---------- Table 1 goes about here ----------

However, the picture changes dramatically in the other bifurcation scenarios. The next two panels show the emergence of a supercritical pitchfork bifurcation. If f is about 0.933, the fundamental steady state loses its local asymptotical stability and two nonfundamental steady states appear in its place. The two bifurcation diagrams only differ with respect to the chosen initial conditions. Note that housing prices may persistently be higher (second panel) or lower (third panel) than the fundamental steady state. If f increases further, we observe cyclical or even chaotic price dynamics restricted to either the bull or the bear market. For f larger than about 4.5, we find that housing prices endogenously switch between bull and bear market regions (we will discuss this phenomenon in further detail in the next subsection with the help of figure 3).

---------- Figure 1 goes about here ----------

The bottom panel depicts a supercritical Neimark-Sacker bifurcation. As f exceeds the value of 0.953, the fundamental steady state becomes unstable and instead we observe quasi-periodic motion. Note that the amplitude of the price fluctuations increases with f. The bifurcation diagram also reveals some periodic windows and areas where the dynamics is apparently chaotic (the latter feature will also be revisited in the next subsection, jointly with figure 4).

It is also instructive to represent the region of local asymptotic stability of the fundamental steady state in the plane of the parameters (c, f) by taking the supply parameter e and the depreciation parameter $(1-d)$ as given. Parameters c and f are particularly important since our analysis stresses the joint effect of real and speculative demand. Note first that each of the three inequalities (19), (20), and (21) results in a half-plane in (c, f) parameter space. The straight lines which bound these half-planes have identical slopes but different intercepts. We can thus easily identify two possible qualitative cases, which we denote as “Case 1” and “Case 2” in figure 2. Since $e > 0$ and $0 < d < 1$, the inequalities $\frac{e}{1-d} > \frac{e}{1+d} - 2$, $\frac{e}{1-d} > 0$ and $\frac{1}{d} - 1 > 0$ always hold. In the qualitative sketches of “Case 1” and “Case 2” it is assumed that parameters *e* and *d* are selected in such a way that $e < 2(1+d)$, i.e. $\frac{e}{1+d} - 2 < 0$. The pictures would also remain qualitatively the same in the case $0 \le \frac{e}{1+d} - 2 < \frac{1}{d} - 1$, except that now the bottom line would lie entirely in the positive quadrant.

However, the qualitative situation $e < 2(1+d)$ is particularly informative. In this case, an interval of positive values of parameter c exists such that (given the selected value of parameter e) the steady state of the model without speculation is stable. Such an interval, given as $(0, 2 - e/(1+d))$, is represented in bold on the horizontal axis. In the opposite case, the model without speculative demand would be unstable for any value of c.

---------- Figure 2 goes about here ----------

Let us now compare “Case 1” with “Case 2”. The bifurcation scenario sketched in “Case 1” occurs when the following condition (which is easily interpreted graphically) holds

$$\frac{e}{1-d} > \frac{1}{d} - 1, \tag{22}$$

or, equivalently, $e > \delta^2/(1-\delta)$, where $\delta := 1-d$ is the depreciation rate. “Case 1” occurs, therefore, if the depreciation rate is small enough. In “Case 1” we observe a Neimark-Sacker bifurcation if the extrapolating component of the demand (governed by parameter f) is sufficiently strong. However, it is also possible for a (subcritical) flip bifurcation to occur if parameter f becomes small enough (assuming that parameter c is outside the range of stability of the model with no speculative demand). The latter bifurcation can be regarded as the possibility that a sufficiently strong component of extrapolative demand stabilizes an otherwise unstable steady state via a reverse subcritical flip bifurcation.

The above considerations about the Flip bifurcation also remain true in "Case 2". However, in this scenario (which occurs if $e < \delta^2 /(1-\delta)$, i.e. when the rate of depreciation is sufficiently high), there is no Neimark-Sacker bifurcation but a pitchfork bifurcation occurs instead when the speculative demand becomes strong enough. The latter gives rise to two further locally stable nonfundamental steady states.

3.3 Some numerical examples

The goal of this subsection is to study the types of dynamic behavior our model may produce in greater detail. In particular, we will investigate two examples. The first example, given in figure 3, corresponds to the pitchfork bifurcation scenario depicted in the second and third panel of figure 1. In figure 3, we now assume $f = 5$. The top panel shows housing prices in deviations from their fundamental value, whereas the bottom panel presents the stock of houses, also in terms of deviations from the fundamental steady state. As can be seen, our model is able to generate complex bull and bear market dynamics. Both housing prices and the stock of houses may fluctuate in an intricate manner for some time above their long-run steady state values. Then, however, out of the blue, housing markets crash, after which both variables fluctuate below their steady state values. Note that the duration of bull and bear market episodes is quite unpredictable.

---------- Figure 3 goes about here ----------

A second example of intricate housing price cycles is given in figure 4. The underlying parameter setting is that used in the bottom panel of figure 1 with $f = 6$, i.e. after the Neimark-Sacker bifurcation. Now the dynamics is characterized by irregular bubbles and crashes. Housing prices may increase for a number of periods. At some point, however, a correction sets in, which usually leads to a severe crash. It is interesting to note that the model is able to generate boom and bust cycles with quite different appearances. Both the duration and amplitude of the cycles vary considerably. This is also mirrored in the development of the stock of houses.

---------- Figure 4 goes about here ----------

Recall that real home prices in London more than doubled from 1983 to 1988 and then fell 47 percent by 1996. From 1996 to 2008, real home prices in London nearly tripled again. However, the latter development was briefly interrupted between mid-2004 to mid-2005, when real home prices decreased by about 6 percent. This downturn was then quickly reversed with annual growth rates of 9 percent. According to Shiller (2007b), such irregularities in boom and bust cycles are hard to explain with standard economic thinking since one would expect a price dip to mark the end of a bubble and lead directly to a crash. We find it worthwhile to point out that our model may endogenously generate such price dynamics.

In the panels of figure 5, we present from top to bottom π_t versus ζ_t, π_t versus ζ_{t-1}, and π_t versus π_{t-1}, respectively. The left-hand panels are based on the dynamics of figure 3 while the right-hand panels show the same for the dynamics of figure 4. The appearance of strange attractors underlines the complexity of the dynamics our model is able to produce. However, these panels also indicate a number of striking differences between the dynamics of figures 3 and 4. In all three panels on the left-hand side, we can make out a positive relation between the plotted variables, that is, we observe that prices tend to increase with the current and previous period's stock of houses and that prices display some kind of persistence (i.e. high prices tend to be followed by high prices, and likewise for low prices). With respect to the persistence of prices, we find a similar effect on the right-hand side. However, the relation between the price of houses and stock of houses is negative for the dynamics discussed in figure 4.

---------- Figure 5 goes about here ----------

Let us finally try to sketch the events that may drive housing price bubbles. Suppose, for instance, that prices are slightly above the fundamental value. Then the majority of agents is optimistic and expects a price increase. As a result, demand for houses increases and prices are pushed upwards for a certain period. During this process, however, the market appears to be increasingly overvalued and agents start to switch to mean reversion expectations. Then some kind of adjustment towards the fundamental value sets in. If this adjustment is rather strong, we may even observe a crash. Otherwise, the rally continues after the price dip. Of course, the real part of the model also impacts on the dynamics. As long as housing prices are high, new constructions increase the stock of houses. During a downwards movement, however, the demand for houses may be considerably lower than the supply of houses, amplifying any price reduction. This story is in line with the conclusion of Shiller (2008), who argues that there has been a tendency in many cities for home prices to rise and crash, but to show little long-term trend. Prices rise while people are optimistic, but forces are set in motion for them to crash when they get too high. In our model, these forces contain a speculative component (dominance of regressive expectations) as well as a real component (excess supply of houses).

4 Conclusions

In this paper we develop a simple model of a speculative housing market to improve our understanding of boom and bust housing prices cycles. The key feature of our model is that the demand for houses is affected by speculative forces. While some agents are convinced that housing prices converge towards their long-run fundamental value, other agents optimistically (pessimistically) believe in the persistence of bull and bear market dynamics. Since agents change their prediction strategies from time to time with respect to market circumstances, our model is nonlinear. We find that such speculative forces may imply the coexistence of (strange) attractors, and can lead to complex price dynamics. In particular, our model has the potential to generate intricate bubbles and crashes, as observed recently in many housing price markets around the world.

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Appendix

In this appendix, we derive the two-dimensional nonlinear dynamical system of the full model, its fixed points, the parameter space for which the model's fundamental steady state is locally asymptotically stable, and necessary conditions for the emergence of a flip, a pitchfork, and a Neimark-Sacker bifurcation, respectively. A theoretical treatment of linear and nonlinear dynamical systems is provided by Gandolfo (2002) and Medio and Lines (2001), among others.

Note first that, using the auxiliary variables $\pi_t = P_t - \overline{P}$ and $\zeta_t = Z_t - \overline{Z}$, the two-dimensional linear dynamical system for the model without speculation

$$\begin{cases} P_{t+1} = (1-c-e)P_t - dZ_t + b \\ Z_{t+1} = eP_t + dZ_t \end{cases} \tag{A1}$$

may be rewritten in terms of deviations from the fundamental steady state as

$$\begin{cases} \pi_{t+1} = (1-c-e)\pi_t - d\zeta_t \\ \zeta_{t+1} = e\pi_t + d\zeta_t \end{cases}. \tag{A2}$$

By now including the speculative demand term, we easily obtain the nonlinear map

$$\begin{cases} \pi_{t+1} = (1-c-e)\pi_t + \dfrac{f\pi_t - gh\pi_t^3}{1+h\pi_t^2} - d\zeta_t \\ \zeta_{t+1} = e\pi_t + d\zeta_t \end{cases}. \tag{A3}$$

Hence, the dynamics of our model is driven by the iteration of a first-order system in (π_t, ζ_t).

By inserting $(\pi_{t+1}, \zeta_{t+1}) = (\pi_t, \zeta_t) = (\overline{\pi}, \overline{\zeta})$ into (A3), the fixed points

$$(\overline{\pi}_1, \overline{\zeta}_1) = (0,0) \tag{A4}$$

and

$$(\bar{\pi}_{2,3}, \bar{\zeta}_{2,3}) = \left(\pm\sqrt{\frac{(1-d)(f-c)-e}{h(e+(1-d)(c+g))}}\;, \quad \frac{e}{1-d}\bar{\pi}_{2,3} \right) \tag{A5}$$

can be calculated. Since the denominator of $\bar{\pi}_{2,3}$ is always positive, the fixed points $(\bar{\pi}_{2,3}, \bar{\zeta}_{2,3})$ only exist if $(1-d)(f-c)-e>0$.

The Jacobian matrix of our model, evaluated at the steady state $(\bar{\pi}_1, \bar{\zeta}_1) = (0,0)$, reads

$$J = \begin{pmatrix} 1-c-e+f & -d \\ e & +d \end{pmatrix}, \tag{A6}$$

where $tr = 1-c-e+d+f$ and $\det = d(1-c+f)$ stand for the trace and determinant of J, respectively. Necessary and sufficient conditions which guarantee that a fixed point of a two-dimensional nonlinear map is locally asymptotically stable are (i) $1+tr+\det>0$, (ii) $1-tr+\det>0$ and (iii) $1-\det>0$, respectively. After some simple transformations, this yields

$$f > c + \frac{e}{1+d} - 2, \tag{A7}$$

$$f < c + \frac{e}{1-d}, \tag{A8}$$

and

$$f < c + \frac{1}{d} - 1. \tag{A9}$$

Observe that for $f=0$, (A7) to (A9) are identical to (9) to (11). In this case, (A8) and (A9) would always be fulfilled. For $f>0$, however, (A7) is less restrictive than (9), while (A8) and (A9) impose additional stability restrictions. Note also that (A7)-(A9) are independent of parameters b and h.

Violation of the first, second and third inequality (the remaining two inequalities hold) represents a necessary condition for the emergence of a flip, pitchfork and Neimark-Sacker bifurcation, respectively. In connection with supporting numerical evidence, this is usually regarded as strong evidence. Figure 1 furthermore reveals that the flip bifurcation is of the subcritical case whereas the pitchfork and Neimark-Sacker bifurcations are of the supercritical type.

Scenario	*c*	*d*	*e*	*f*	*g*	*h*
Flip	0.1	0.5	3	-	1	1
Pitchfork	0.6	0.7	0.1	5	1.5	1
Neimark-Sacker	0.9	0.95	0.5	6	0.3	1

Table 1: Parameter settings for numerical results.

Captions for figures

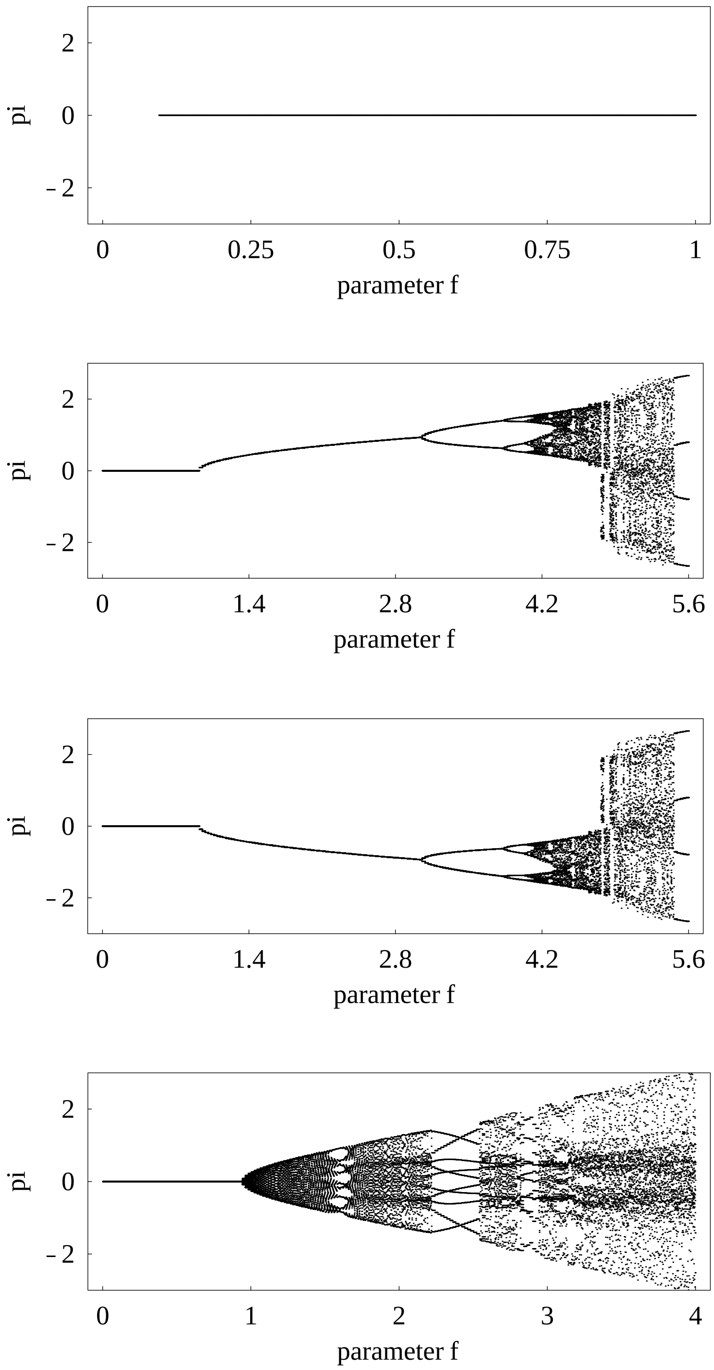
Figure 1: Bifurcation scenarios. The first panel shows a (subcritical) flip bifurcation, the second and third panels show a (supercritical) pitchfork bifurcation for two different sets of initial conditions, and the bottom panel shows a Neimark-Sacker bifurcation. Parameter setting as in table 1, except that parameter f is varied as indicated on the axis.

Figure 2: Representations of local asymptotic stability regions of the 'fundamental steady state' in the plane of the parameters (c, f), taking supply parameter e and depreciation parameter $(1-d)$ as given. The left (right) panel depicts "Case 1" ("Case 2"), i.e. a situation where the rate of depreciation is relatively low (high).

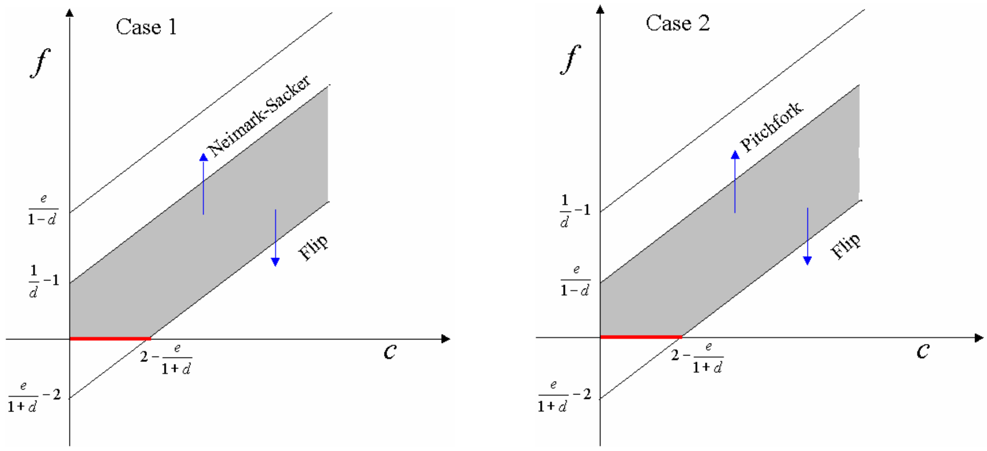
Figure 3: Examples of persistent bull and bear market dynamics. The top panel shows the evolution of housing prices and the bottom panel presents the development of the stock of houses (both in deviations from the fundamental steady state). The parameter setting corresponds to the pitchfork bifurcation scenario, as indicated in table 1.

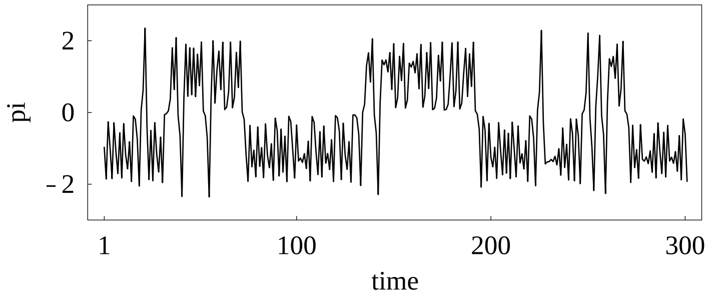
Figure 4: Examples of bubbles and crashes. The top panel shows the evolution of housing prices and the bottom panel presents the development of the stock of houses (both in deviations from the fundamental steady state). The parameter setting corresponds to the Neimark-Sacker bifurcation scenario, as indicated in table 1.

Figure 5: Emergence of strange attractors. In the panels from top to bottom, we plot π_t versus ζ_t, π_t versus ζ_{t-1}, and π_t versus π_{t-1}, respectively. The left-hand panels are based on the dynamics of figure 2, whereas the right-hand panels belong to the dynamics reported in figure 3.



pi
2
0
- 2
0
0.25
0.5
0.75
1
parameter f
pi
2
0
- 2
0
1.4
2.8
4.2
5.6
parameter f
pi
2
0
- 2
0
1.4
2.8
4.2
5.6
parameter f
pi
2
0
- 2
0
1
2
3
4
parameter f


Case 1
f
c
Neimark-Sacker
Flip
$\frac{e}{1-d}$
$\frac{1}{d}-1$
$\frac{e}{1+d}-2$
$2-\frac{e}{1+d}$
Case 2
f
c
Pitchfork
Flip
$\frac{1}{d}-1$
$\frac{e}{1-d}$
$\frac{e}{1+d}-2$
$2-\frac{e}{1+d}$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- 2
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100
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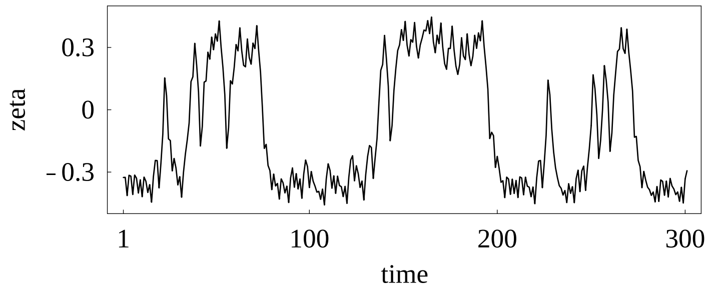
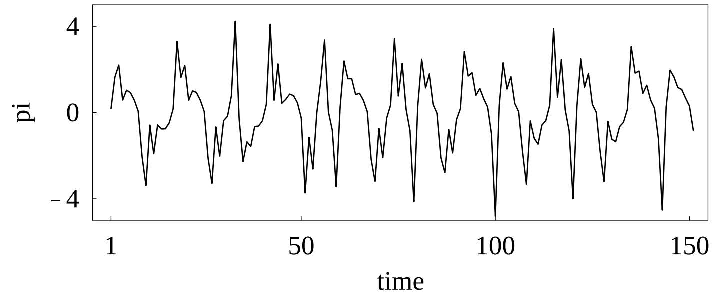

0.3
0
- 0.3
zeta
1
100
200
300
time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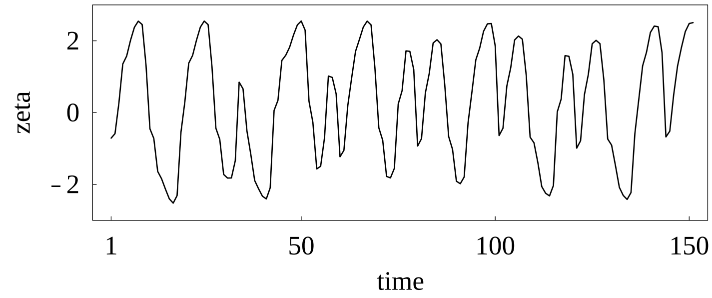
- 4
pi
1
50
100
150
time


2
0
- 2
zeta
1
50
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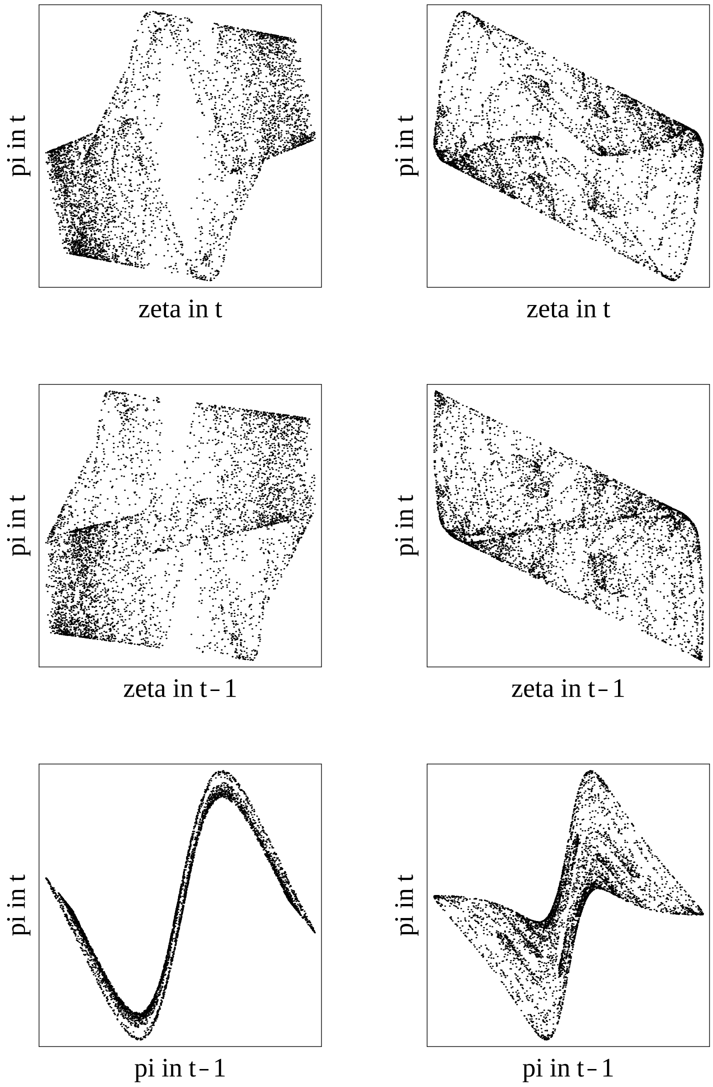
150
time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zeta in t
pi in t
zeta in t
pi in t
zeta in t−1
pi in t
zeta in t−1
pi in t
pi in t−1
pi in t
pi in t−1

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