Resurrecting Laplace's Demon: The Case for Deterministic Models

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Invited Talk: Synchron
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Context: Cyber-Physical Systems

A particularly challenging case for determinism

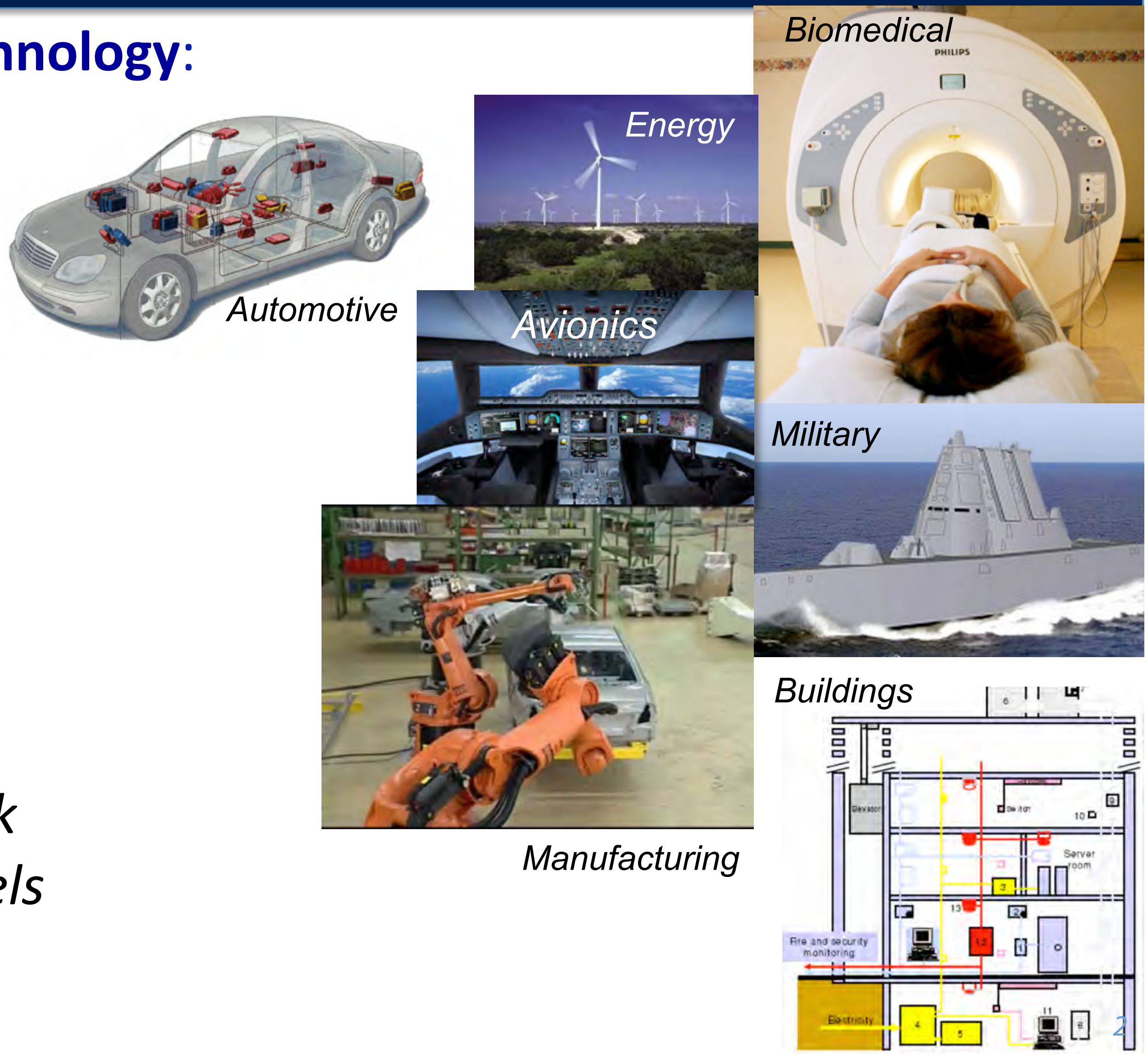
Not just information technology:

- Cyber + *Physical*
- Computation + *Dynamics*
- Security + *Safety*

Properties:

- Highly dynamic
- Safety critical
- Uncertain environment
- Physically distributed
- Sporadic connectivity
- Resource constrained

Does it make sense to talk about deterministic models for such systems?



Models vs. Reality

$$x(t) = x(0) + \int_0^t v(\tau) d\tau$$
$$v(t) = v(0) + \frac{1}{m} \int_0^t F(\tau) d\tau.$$

The model

The target (the thing being modeled).

In this example, the *modeling framework* is calculus and Newton's laws.

Fidelity is how well the model and its target match

Engineers often confuse the model with its target

You will never strike oil by drilling through the map!

Solomon Wolf Golomb

But this does not in any way diminish the value of a map!

Lee, Berkeley

Determinacy

Some of the most valuable models are *deterministic*.

A model is *deterministic* if, given the initial *state* and the *inputs*, the model defines exactly one *behavior*.

Deterministic models have proven extremely valuable in the past.

Laplace's Demon

"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes."

— Pierre Simon Laplace

Pierre-Simon Laplace (1749–1827).
Portrait by Joan-Baptiste Paulin Guérin, 1838

Did quantum mechanics dash this hope?

"At first, it seemed that these hopes for a complete determinism would be dashed by the discovery early in the 20th century that events like the decay of radioactive atoms seemed to take place at random. It was as if God was playing dice, in Einstein's phrase. But science snatched victory from the jaws of defeat by moving the goal posts and redefining what is meant by a complete knowledge of the universe."

(Stephen Hawking, 2002)

Nevertheless, Laplace's Demon cannot exist.

In 2008, David Wolpert, then at NASA, now at the Santa Fe Research Institute, used Cantor's diagonalization technique to prove that Laplace's demon cannot exist. His proof relies on the observation that such a demon, were it to exist, would have to exist in the very physical world that it predicts.

David Wolpert

The Koptez Principle

Hermann Kopetz
Professor (Emeritus)
TU Vienna

Many properties that we assert about systems (determinism, timeliness, reliability) are in fact not properties of the system, but rather properties of a *model* of the system.

If we accept this, then it makes no sense to talk about whether the physical world is deterministic. It only makes sense to talk about whether *models* of the physical world are deterministic.

The question switches from whether a model is *True* to whether it is *Useful*

"Essentially, all models are wrong,
but some are useful."

Box, G. E. P. and N. R. Draper, 1987: *Empirical Model-Building and Response Surfaces*. Wiley Series in Probability and Statistics, Wiley.

Physicists continue to debate whether the world is deterministic

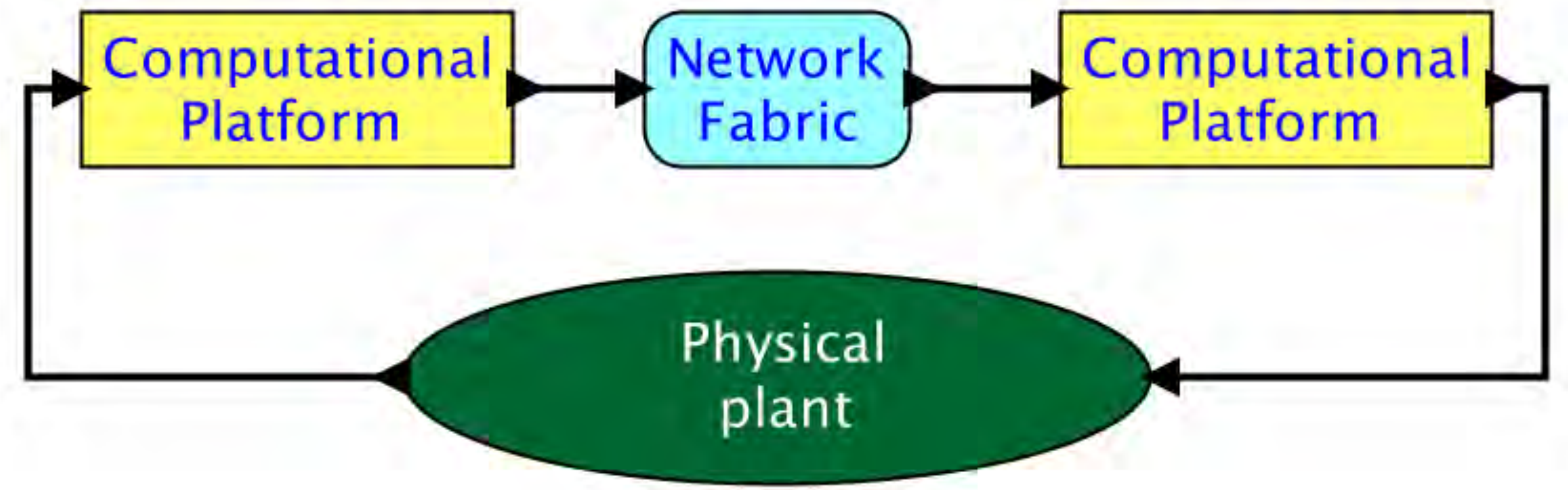
$$
\begin{aligned}
x(t) &= x(0) + \int_0^t v(\tau)d\tau \\
v(t) &= v(0) + \frac{1}{m}\int_0^t F(\tau)d\tau.
\end{aligned}
$$

Deterministic model

Deterministic system?

Determinism is a property of models, not a property of the systems they model.

Schematic of a simple Cyber-Physical System



What kinds of models should we use?

Let's look at the most successful kinds of models from the cyber and the physical worlds.

Software is a Model

Physical System

Model


```java
/** Reset the output receivers, which are the inside receivers of
 *  the output ports of the container.
 *  @exception IllegalActionException If getting the receivers fails.
 */
private void _resetOutputReceivers() throws IllegalActionException {
    List<IOPort> outputs = ((Actor) getContainer()).outputPortList();
    for (IOPort output : outputs) {
        if (_debugging) {
            _debug("Resetting inside receivers of output port: "
                    + output.getName());
        }
        Receiver[][] receivers = output.getInsideReceivers();
        if (receivers != null) {
            for (int i = 0; i < receivers.length; i++) {
                if (receivers[i] != null) {
                    for (int j = 0; j < receivers[i].length; j++) {
                        if (receivers[i][j] instanceof FSMReceiver) {
                            receivers[i][j].reset();
                        }
                    }
                }
            }
        }
    }
}
```

Single-threaded imperative programs are deterministic models

Consider single-threaded imperative programs

```
void foo(int32_t x) {
    if (x > 1000) {
        x = 1000;
    }
    if (x > 0) {
        x = x + 1000;
        if (x < 0) {
            panic();
        }
    }
}
```

This program defines exactly one behavior, given the input x.

Note that the modeling framework (the C language, in this case) defines "behavior" and "input."

The target of the model is electrons sloshing around in silicon. It takes time, consumes energy, and fails if dropped in the ocean, none of which are properties of the model.

Software relies on another deterministic model that abstracts the hardware

Physical System

Model

Image: Wikimedia Commons

Integer Register-Register Operations

RISC-V defines several arithmetic R-type operations. All operations read the *rs1* and *rs2* registers as source operands and write the result into register *rd*. The *funct* field selects the type of operation.

rd (31–27)	rs1 (26–22)	rs2 (21–17)	funct10 (16–7)	opcode (6–0)
5	5	5	10	7
dest	src1	src2	ADD/SUB/SLT/SLTU	OP
dest	src1	src2	AND/OR/XOR	OP
dest	src1	src2	SLL/SRL/SRA	OP
dest	src1	src2	ADDW/SUBW	OP-32
dest	src1	src2	SLLW/SRLW/SRAW	OP-32

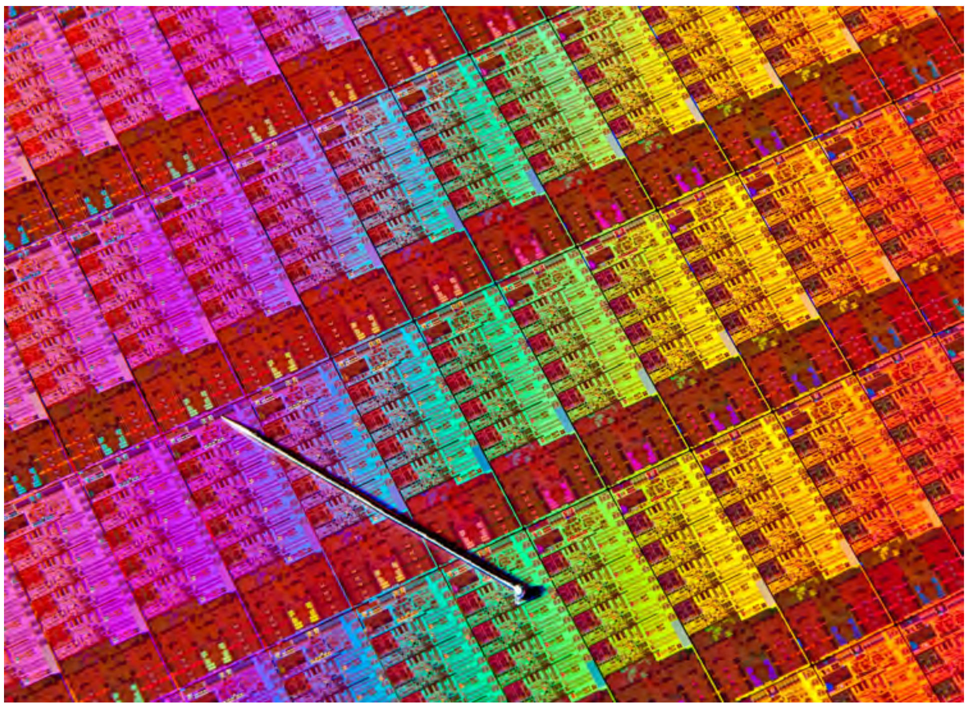
Waterman, et al., The RISC-V Instruction Set Manual, UCB/EECS-2011-62, 2011

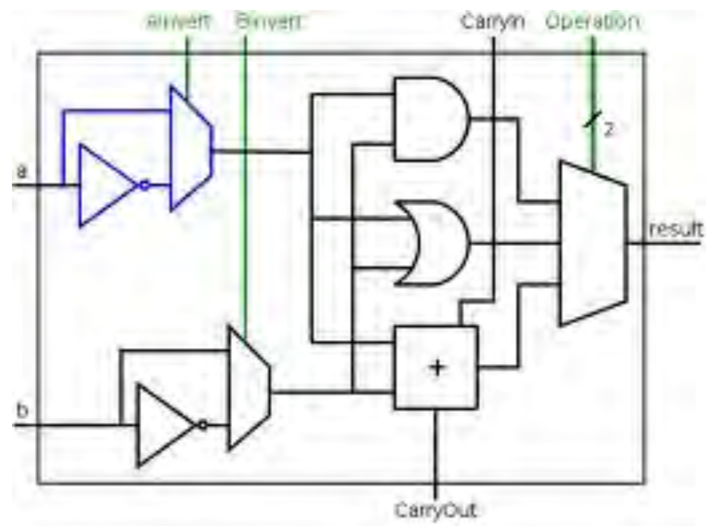
Instruction Set Architectures (ISAs) are deterministic models

… which relies on yet another deterministic model

Physical System

Model





Synchronous digital logic is a deterministic model

Deterministic Models for the Physical Side of CPS

Physical System

Image: Wikimedia Commons

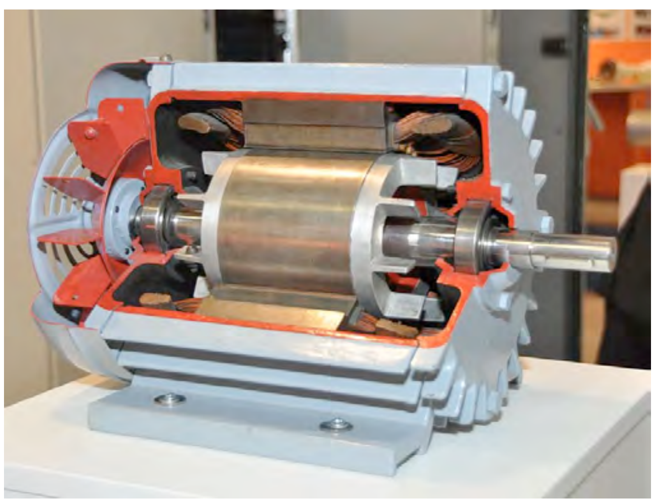
Model

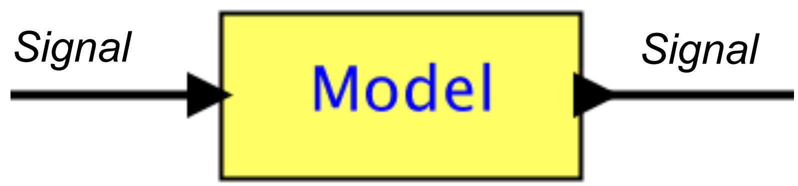
$$\dot{\mathbf{x}}(t) = \dot{\mathbf{x}}(0) + \frac{1}{M}\int_0^t \mathbf{F}(\tau)d\tau$$

Differential Equations are deterministic models

A major problem for CPS: combinations of deterministic models are nondeterministic

```
void initTimer(void) {
    SysTickPeriodSet(SysCtlClockGet() / 1000);
    SysTickEnable();
    SysTickIntEnable();
}
volatile uint timer_count = 0;
void ISR(void) {
    if(timer_count != 0) {
        timer_count--;
    }
}
int main(void) {
    SysTickIntRegister(&ISR);
    .. // other init
    timer_count = 2000;
    initTimer();
    while(timer_count != 0) {
        ... code to run for 2 seconds
    }
    ... // other code
}
```





$$\dot{\mathbf{x}}(t) = \dot{\mathbf{x}}(0) + \frac{1}{M}\int_0^t \mathbf{F}(\tau)\,d\tau$$

Image: Wikimedia Commons

Timing is not part of software and network semantics

Correct execution *of a program in all widely used programming languages, and* ***correct delivery*** *of a network message in all general-purpose networks has nothing to do with how long it takes to do anything.*

Programmers have to step *outside* the programming abstractions to specify timing behavior.

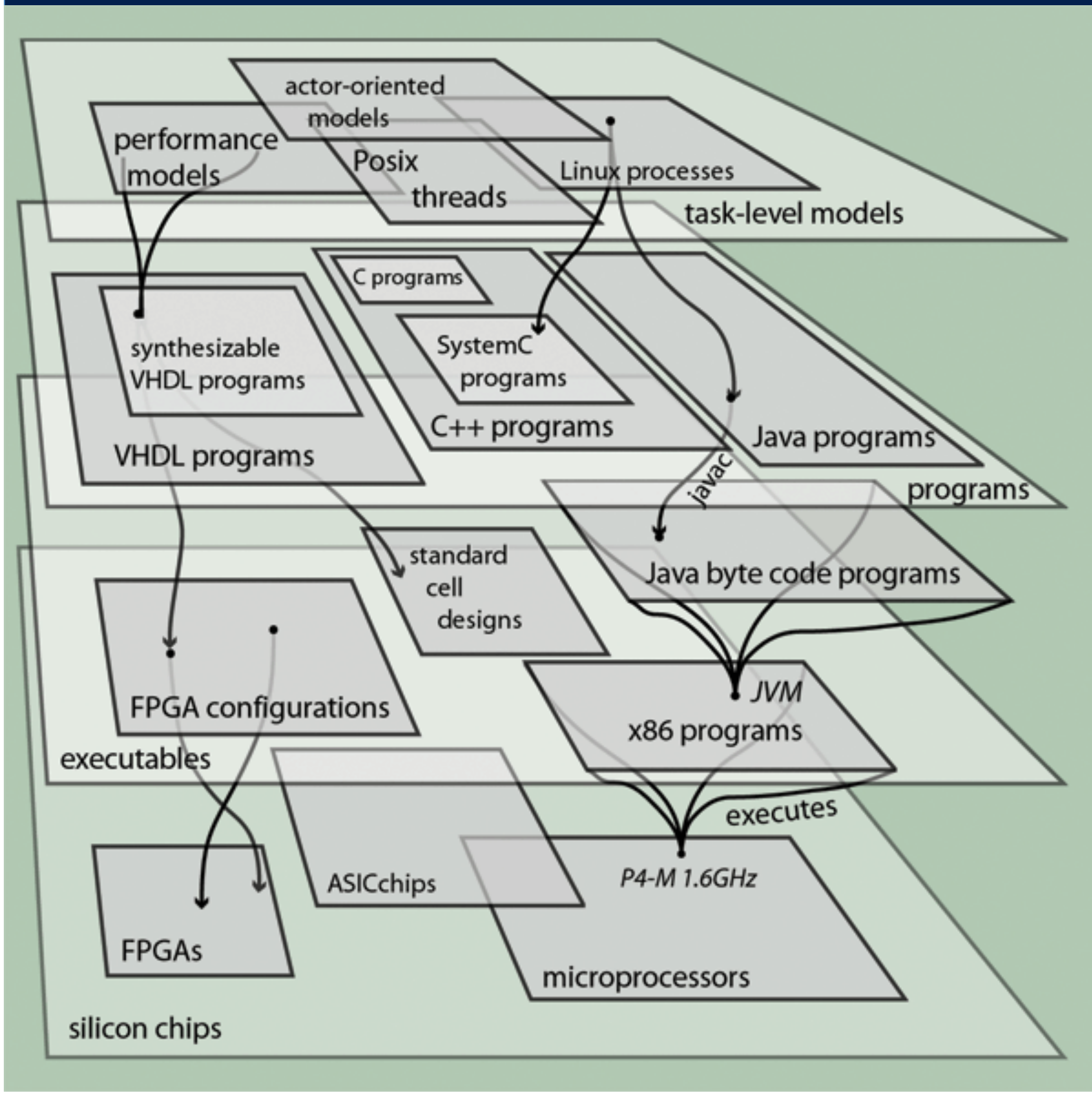
CPS designers have no map!

A Story

In “fly by wire” aircraft, computers control the plane, mediating pilot commands.

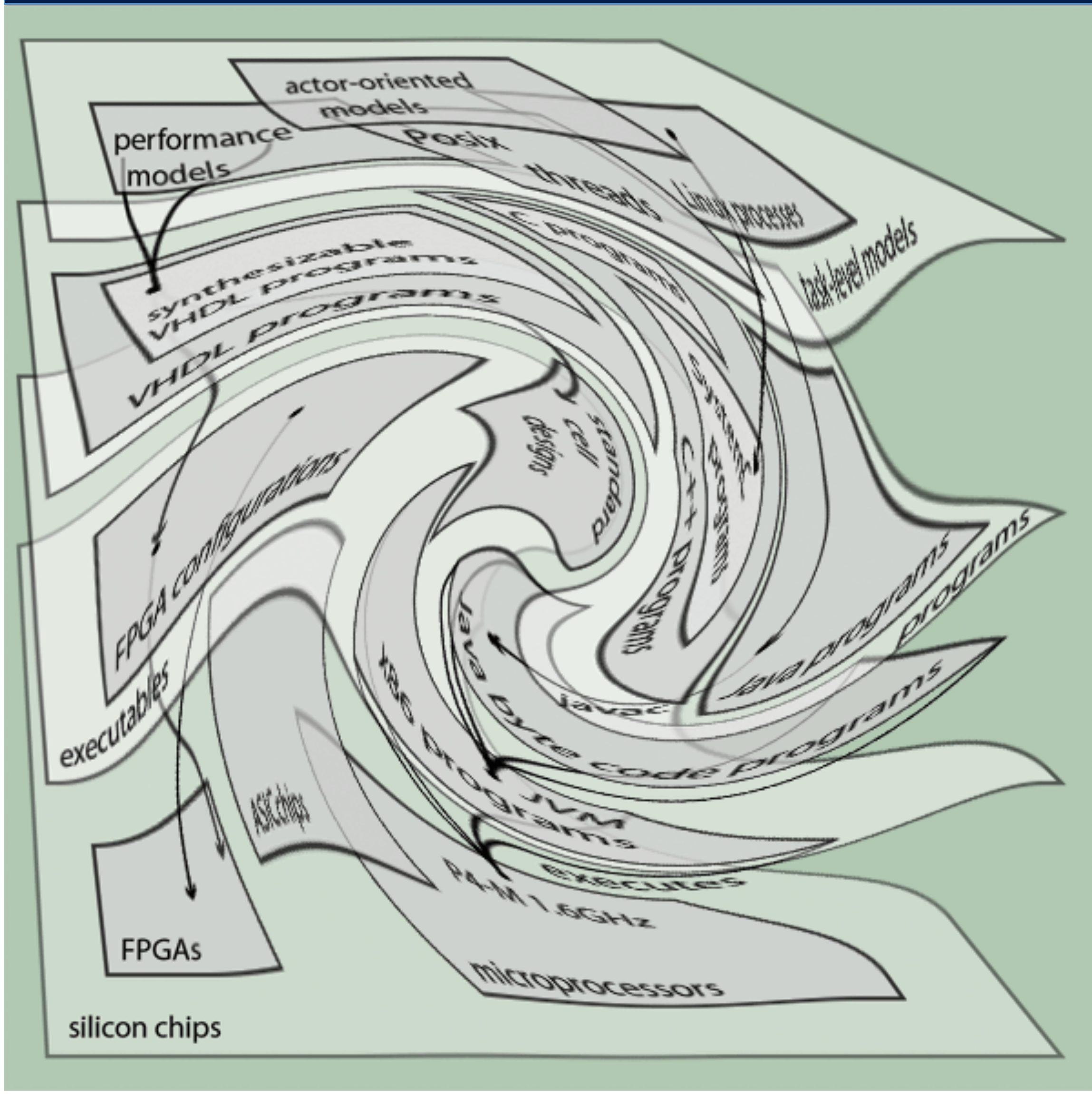
Abstraction Layers

All of which are models except the bottom



The purpose of an abstraction is to hide details of the implementation below and provide a platform for design from above.

Abstraction Layers
All of which are models except the bottom



Every abstraction layer has failed for the aircraft designer.

The design *is* the implementation.

Determinism? Really?

CPS applications operate in an intrinsically nondeterministic world.

Does it really make sense to insist on deterministic models?

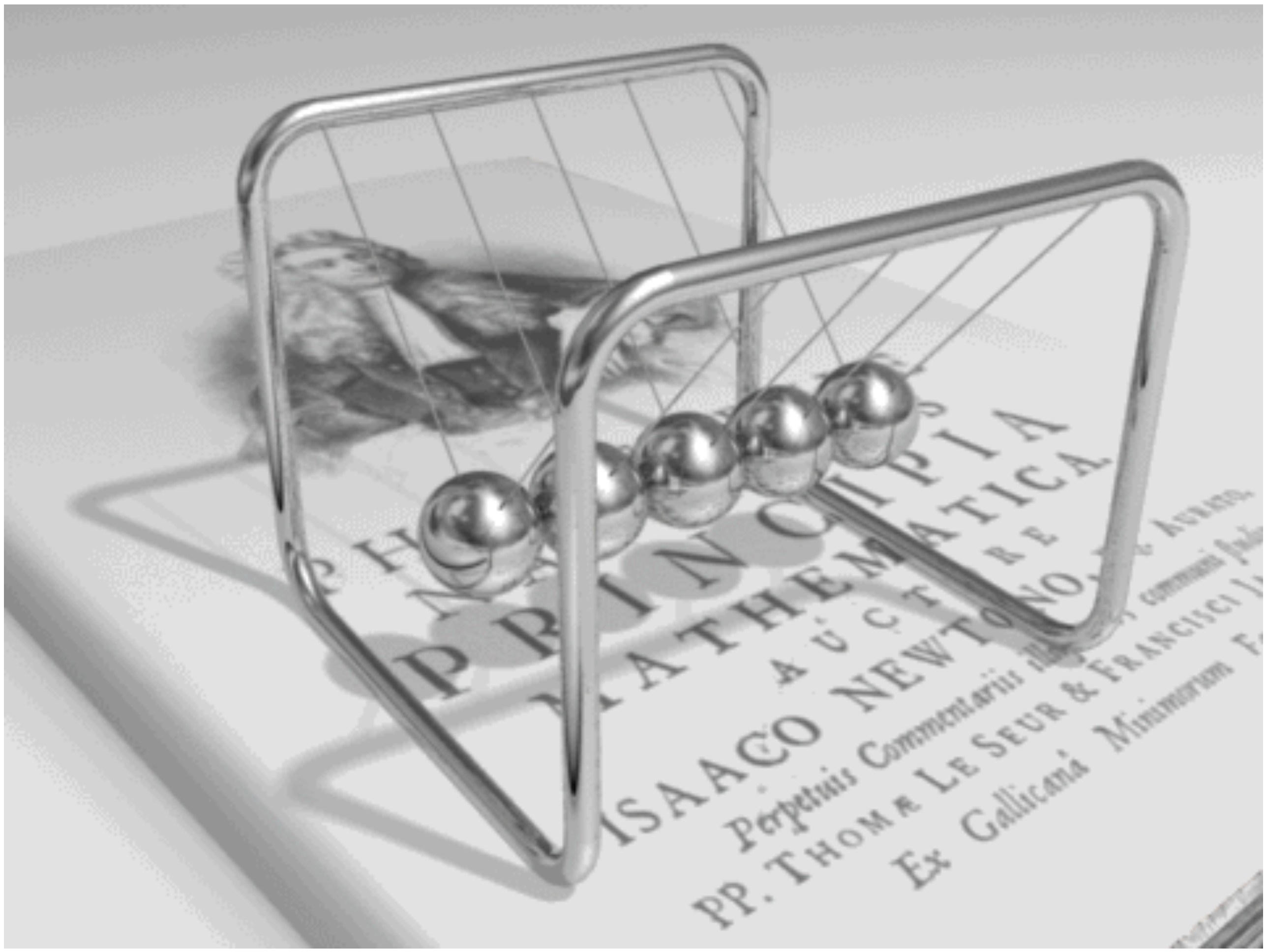
The Value of Models

- In *science*, the value of a *model* lies in how well its behavior matches that of the physical system.
- In *engineering*, the value of the *physical system* lies in how well its behavior matches that of the model.

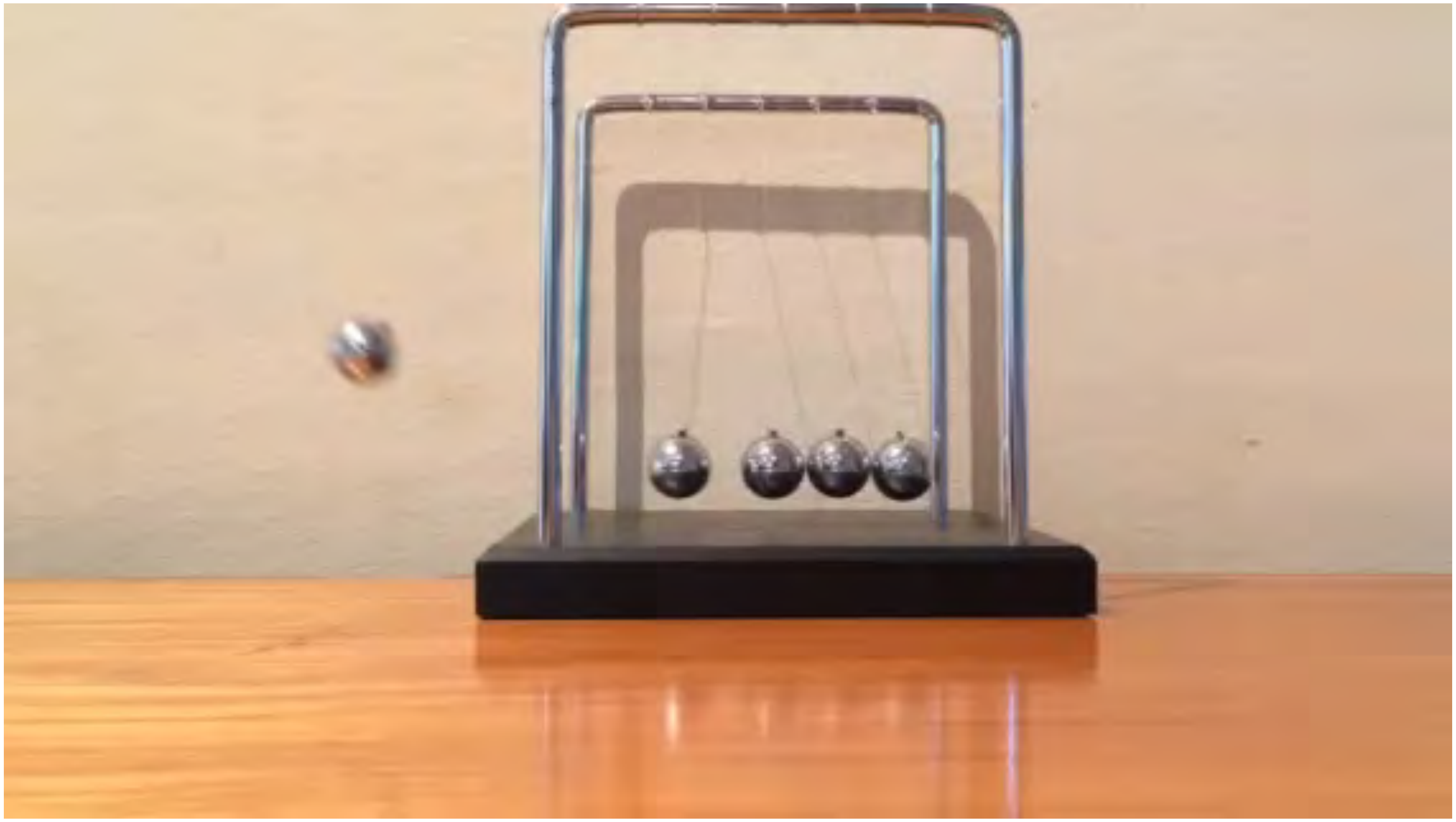
In engineering, model fidelity is a two-way street!

For a model to be useful, it is necessary (but not sufficient) to be able to be able to construct a faithful physical realization.

A Model


PRINCIPIA
MATHEMATICA
ISAACO NEWTONO
Perpetuis Commentariis
PP. THOMÆ LE SEUR & FRANCISCI
Ex Gallicanâ Minimorum

A Physical Realization



Model Fidelity

- To a *scientist*, the model is flawed.
- To an *engineer*, the realization is flawed.

I'm an engineer…

For CPS, we need to change the question

The question is *not* whether deterministic models can describe the behavior of cyber-physical systems (with high fidelity).

The question is whether we can build cyber-physical systems whose behavior matches that of a deterministic model (with high probability).

Determinism?
What about resilience? Adaptability?

Deterministic models do not eliminate the need for robust, fault-tolerant designs.

In fact, they *enable* such designs, because they make it much clearer what it means to have a fault!

Enter: Synchronous Languages

- Deterministic concurrency

But:

- Time between ticks?
- WCET over *all* reactions?
- Distributed systems?

Lee, Berkeley

Useful deterministic models for CPS

To get deterministic models for CPS with faithful implementations, we can:

1. Use processors with controllable timing (PRET machines).
   – http://chess.eecs.berkeley.edu/pret
2. Extend synchronous languages with a (superdense) model of time
   – Lee and Zheng, EMSOFT 2007
3. Synchronize clocks and create distributed real-time execution (PTIDES)
   – http://chess.eecs.berkeley.edu/ptides

Together, these technologies give a programming model for distributed and concurrent real-time systems that is deterministic in the sense of single-threaded imperative programs, and also deterministic w.r.t. to timing of external interactions.

Extending SR to get DE

- Time to the next tick is determined by time-stamped discrete events.
- At each tick, use a least fixed-point semantics, as usual with synchronous languages.

Leveraging Synchronous Language Principles for Heterogeneous Modeling and Design of Embedded Systems*

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Haiyang Zheng
Department of EECS
University of California, Berkeley

EMSOFT 2007

Ptides – A Robust Distributed Deterministic DE MoC

in Proceedings of the 13th IEEE Real-Time and Embedded Technology and Applications Symposium (RTAS 07) , Bellevue, WA, United States.

A Programming Model for Time-Synchronized Distributed Real-Time Systems

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EECS Department
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Jie Liu
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One Microsoft Way

Edward A. Lee
EECS Department
UC Berkeley

Abstract: Discrete-event (DE) models are formal system specifications that have analyzable deterministic behaviors. Using a global, consistent notion of time, DE components communicate via time-stamped events. DE models have primarily been used in performance modeling and simulation, where time stamps are a modeling property bearing no relationship to real time during execution of the model. In this paper, we extend DE models with the capability of relating certain events to physical time…

Using Synchronized Clocks in Distributed Systems: Roots of the Idea

Using Time Instead of Timeout for Fault-Tolerant Distributed Systems

LESLIE LAMPORT
SRI International

A general method is described for implementing a distributed system with any desired degree of fault-tolerance. Instead of relying upon explicit timeouts, processes execute a simple clock-driven algorithm. Reliable clock synchronization and a solution to the Byzantine Generals Problem are assumed.

Categories and Subject Descriptors: C.2.4 [**Computer-Communications Networks**]: Distributed Systems—*network operating systems*; D.1.3 [**Programming Techniques**]: Concurrent Programming; D.4.1 [**Operating Systems**]: Process Management—*synchronization*; D.4.3 [**Operating Systems**]: File Systems Management—*distributed file systems*; D.4.5 [**Operating Systems**]: Reliability—*fault-tolerance*; D.4.7 [**Operating Systems**]: Organization and Design—*distributed systems*; *real-time systems*

General Terms: Design, Reliability

Additional Key Words and Phrases: Clocks, transaction commit, timestamps, interactive consistency, Byzantine Generals Problem

ACM Transactions on Programming Languages and Systems, 1984.

Google Spanner – A Reinvention

Google independently developed a very similar technique and applied it to distributed databases.

Spanner: Google's Globally-Distributed Database

James C. Corbett, Jeffrey Dean, Michael Epstein, Andrew Fikes, Christopher Frost, JJ Furman, Sanjay Ghemawat, Andrey Gubarev, Christopher Heiser, Peter Hochschild, Wilson Hsieh, Sebastian Kanthak, Eugene Kogan, Hongyi Li, Alexander Lloyd, Sergey Melnik, David Mwaura, David Nagle, Sean Quinlan, Rajesh Rao, Lindsay Rolig, Yasushi Saito, Michal Szymaniak, Christopher Taylor, Ruth Wang, Dale Woodford

Google, Inc.

Abstract

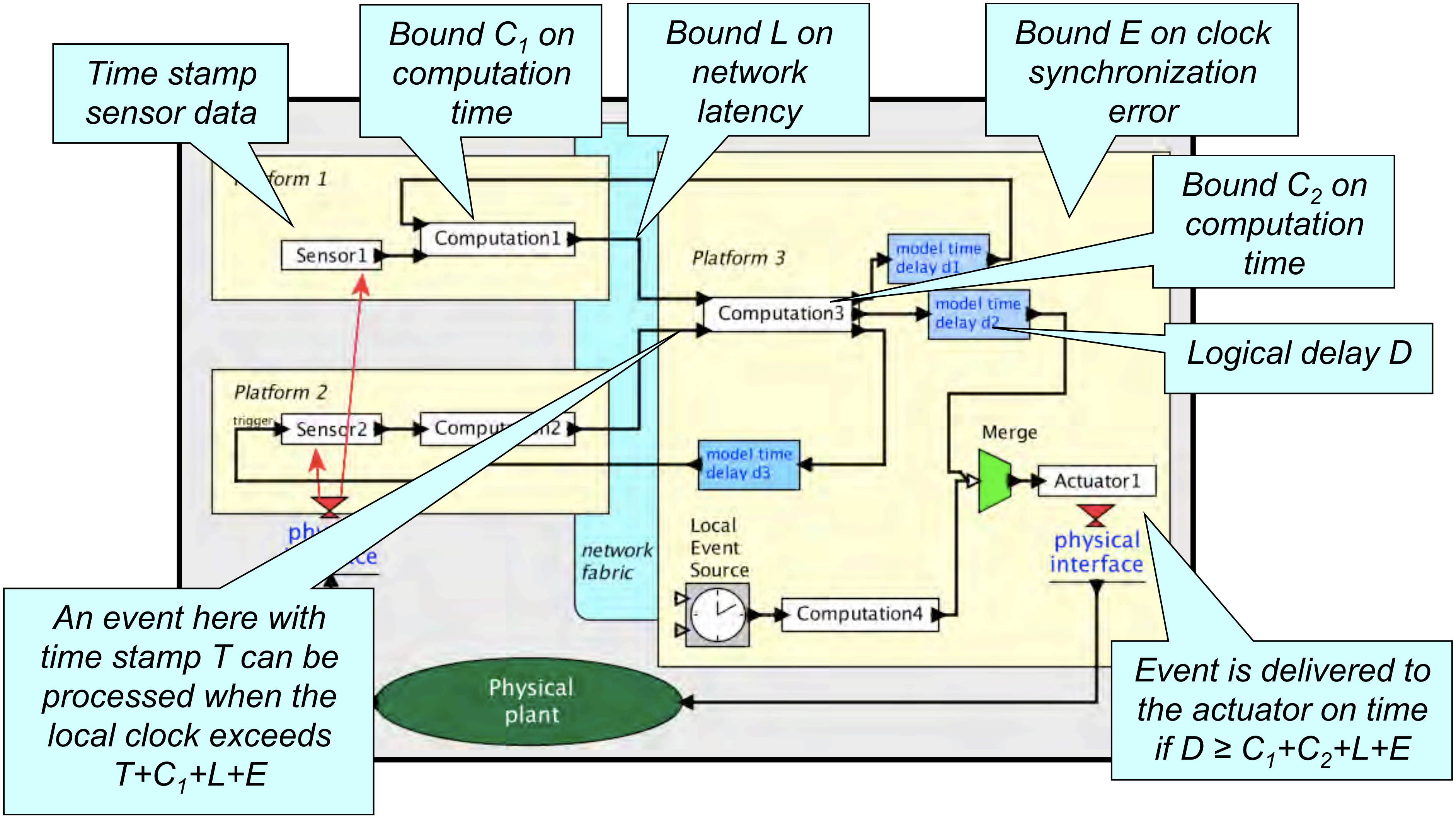
Spanner is Google's scalable, multi-version, globally-distributed, and synchronously-replicated database. It is the first system to distribute data at global scale and support externally-consistent distributed transactions. This paper describes how Spanner is structured, its feature set, the rationale underlying various design decisions, and a novel time API that exposes clock uncertainty. This API and its implementation are critical to supporting external consistency and a variety of powerful features: non-blocking reads in the past, lock-free read-only transactions, and atomic schema changes, across all of Spanner.

tency over higher availability, as long as they can survive 1 or 2 datacenter failures.

Spanner's main focus is managing cross-datacenter replicated data, but we have also spent a great deal of time in designing and implementing important database features on top of our distributed-systems infrastructure. Even though many projects happily use Bigtable [9], we have also consistently received complaints from users that Bigtable can be difficult to use for some kinds of applications: those that have complex, evolving schemas, or those that want strong consistency in the presence of wide-area replication. (Similar claims have been made by other authors [37].) Many applications at Google

Proceedings of OSDI 2012

Ptides: Time stamps bind to real time at sensors and actuators



Deterministic Distributed Real-Time

Assume bounds on:

- *execution time*
- *clock synchronization error*
- *network latency*

then ***events are processed in time-stamp order*** at every component and ***events are delivered to actuators on time***.

See http://chess.eecs.berkeley.edu/ptides

So Many Assumptions?

Non-Synchronized Clocks

You will never strike oil by drilling through the map!

*All of the assumptions are achievable with today's technology, and are **requirements** anyway for hard-real-time systems. The Ptides model makes the requirements explicit.*

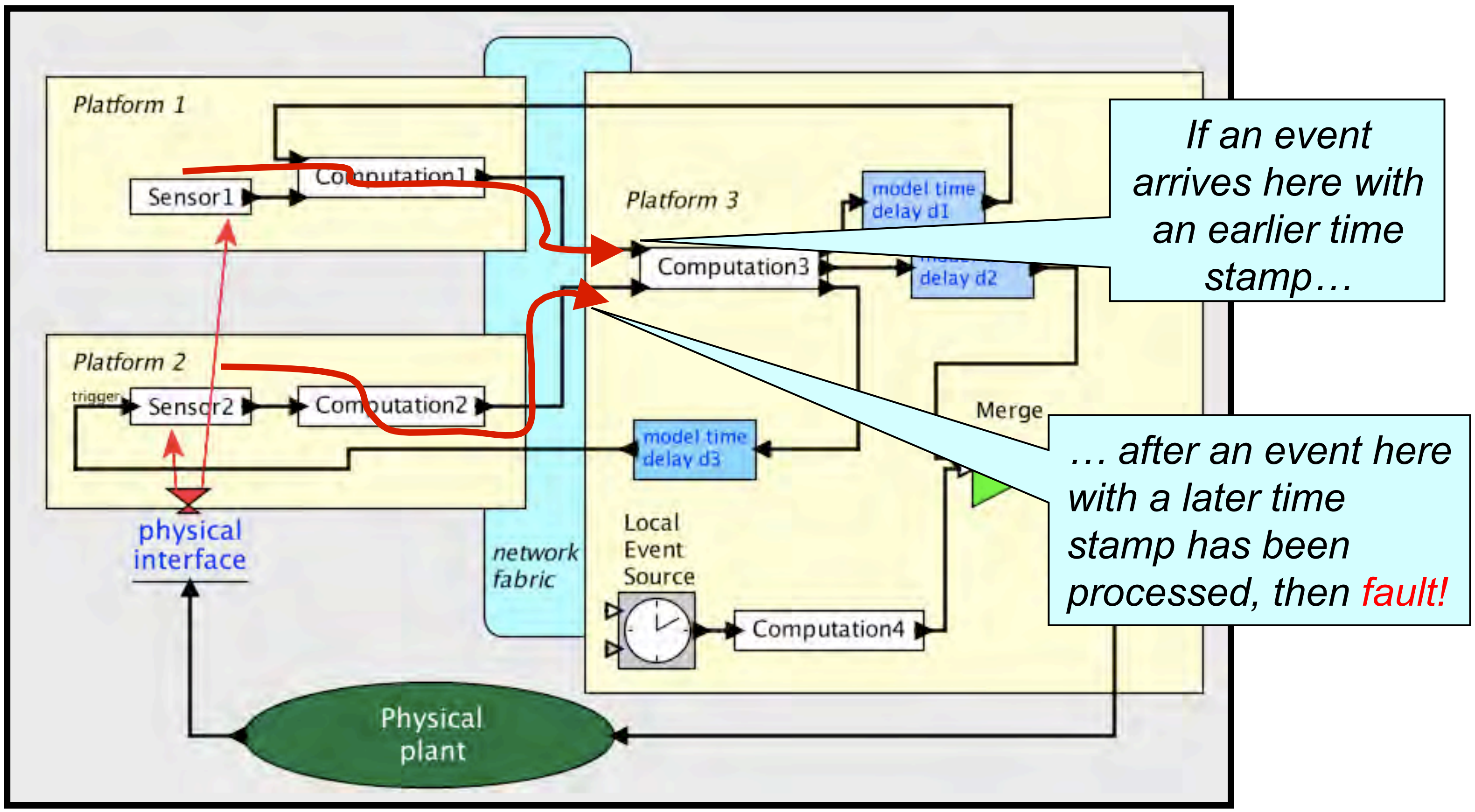
*Violations of the requirements are detectable as out-of-order events and can be treated as **faults**.*

Lee, Berkeley

Handling Faults

A fault manifests as out-of-order events.

Occurrence of a fault implies one or more of the assumptions was violated.



But…

Determinism has its limits.

- Complexity
- Uncertainty
- Chaos
- Incompleteness

Complexity

- Some systems are too complex for deterministic models.

- Nondeterministic abstractions become useful.

“Iron wing” model of an Airbus A350.

Complexity

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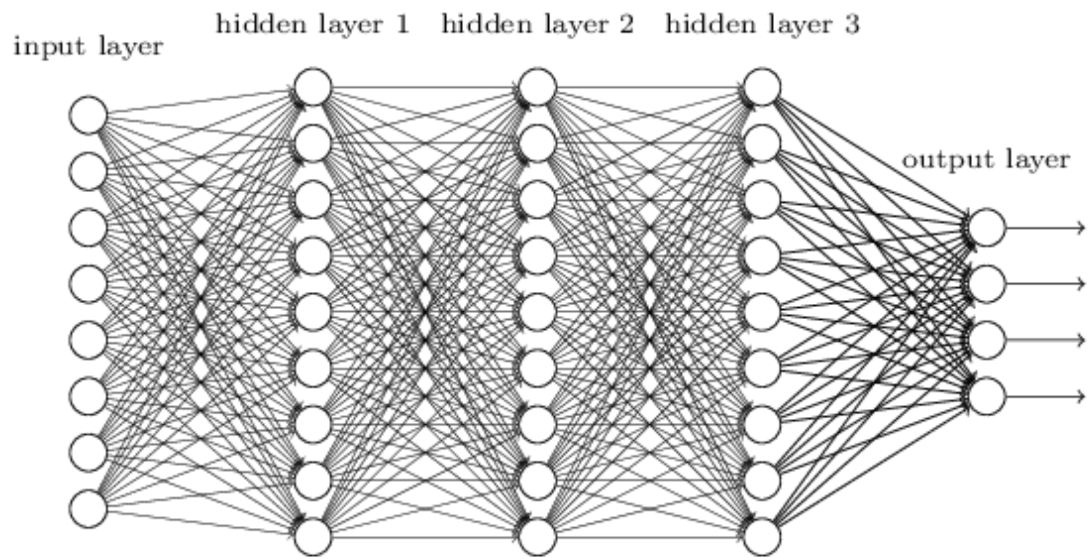

Deep Learning, draft book in preparation, by Yoshua Bengio, Ian Goodfellow, and Aaron Courville. http://www.deeplearningbook.org/

But…

Determinism has its limits.

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Uncertainty

- We can’t construct deterministic models of what we don’t know.
- For this, nondeterminism is useful.
- Bayesian probability (which is mostly due to Laplace) quantifies uncertainty.

Portrait of Reverend Thomas Bayes (1701 - 1761) that is probably not actually him.

But...

Determinism has its limits.

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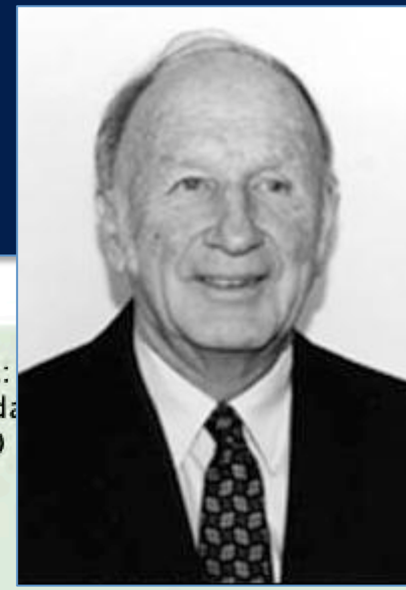
Determinism does not imply predictability

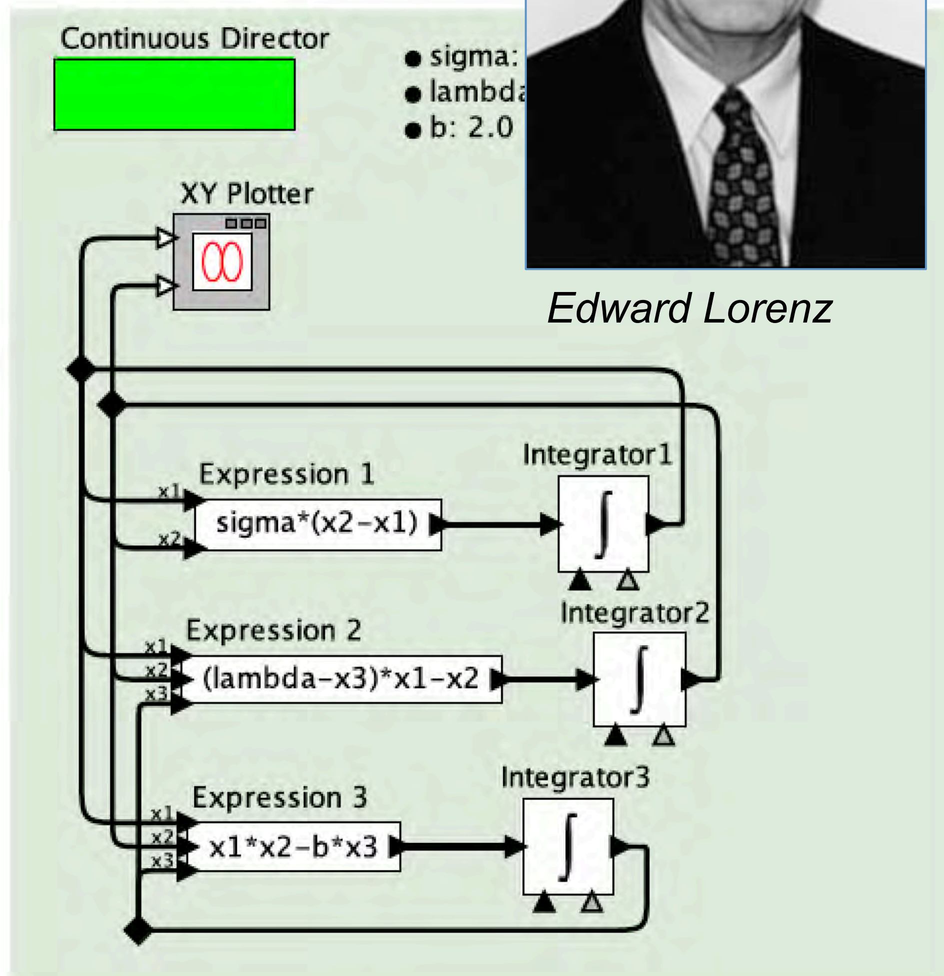


Edward Lorenz

Lorenz attractor:

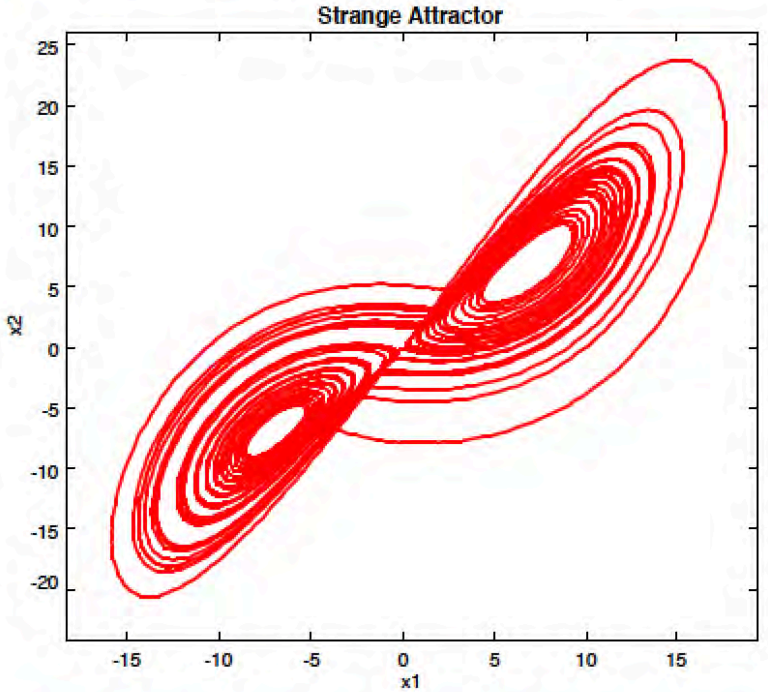
$$\begin{aligned}\dot{x}_1(t) &= \sigma(x_2(t) - x_1(t)) \\ \dot{x}_2(t) &= (\lambda - x_3(t))x_1(t) - x_2(t) \\ \dot{x}_3(t) &= x_1(t)x_2(t) - bx_3(t)\end{aligned}$$

This is a chaotic system, so arbitrarily small perturbations have arbitrarily large consequences.

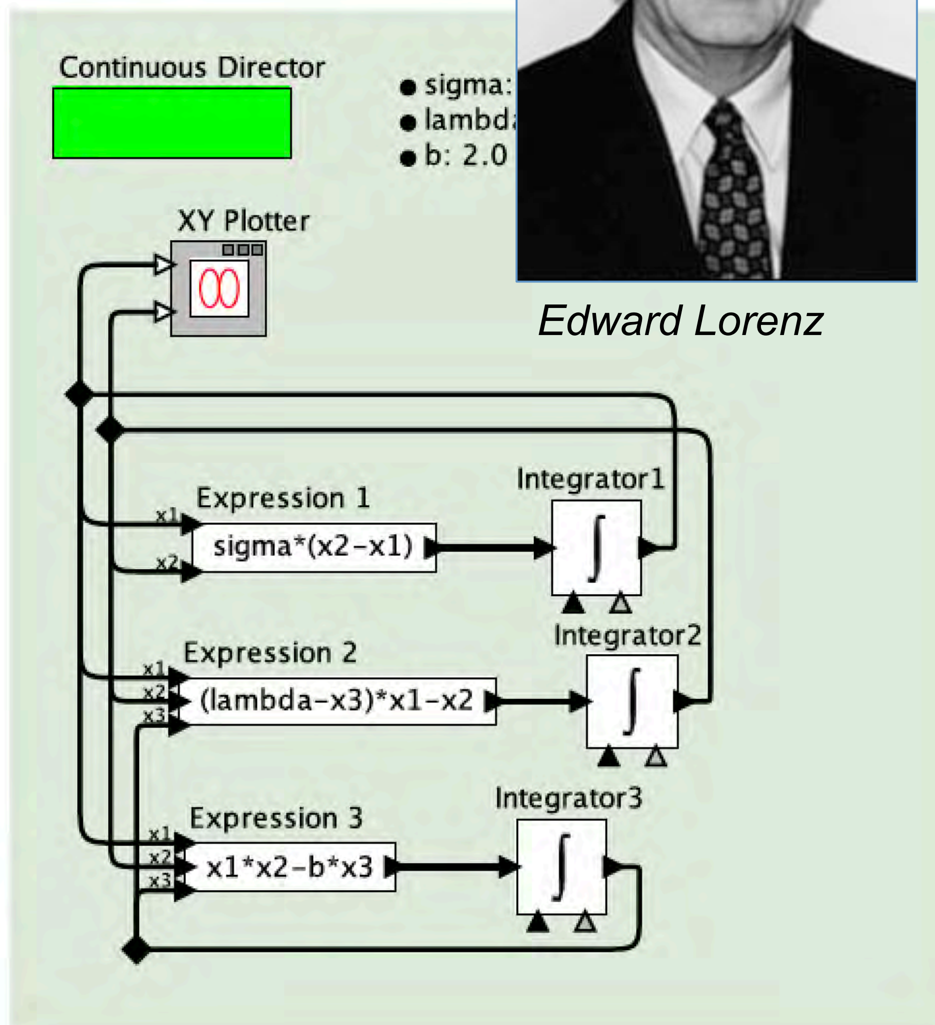


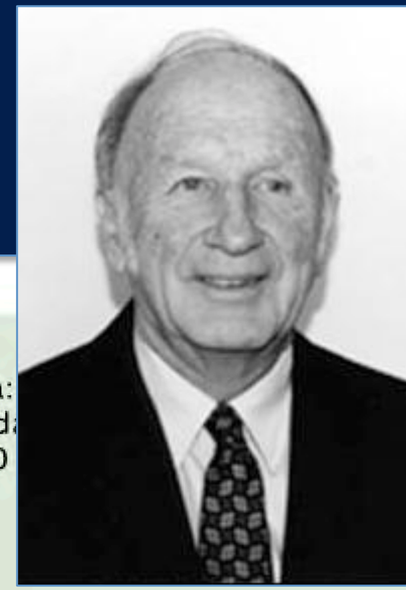
Determinism does not imply predictability

Plot of x_1 vs. x_2:



The position of a point is not meaningfully predictable even though the system is deterministic.





Edward Lorenz

Determinism does not imply predictability

[Thiele and Kumar, EMSOFT 2015]

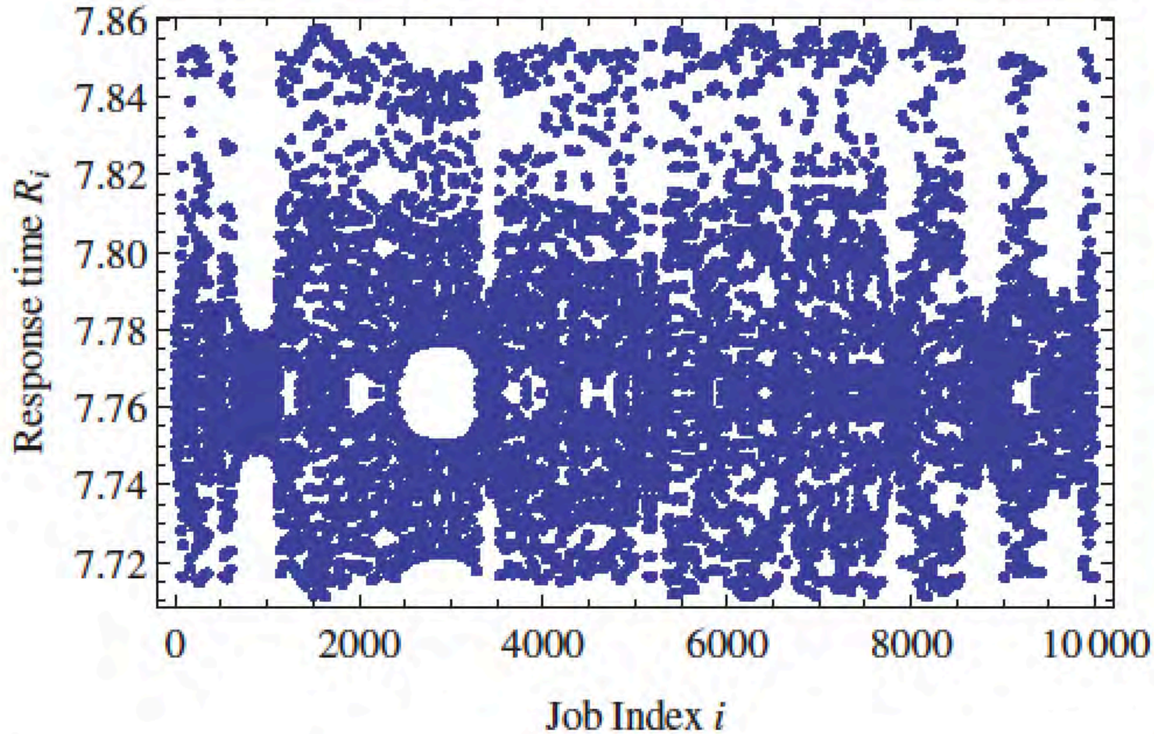


Fig. 15. Response time across jobs for the multi-resource scheduler with $R_s(i-1) = 7.76$ and $R_s(i-2) = 7.74$.

But…

Determinism has its limits.

- Complexity
- Uncertainty
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Incompleteness of Determinism

Any set of deterministic models rich enough to encompass Newton's laws plus discrete transitions is incomplete.

Lee, *Fundamental Limits of Cyber-Physical Systems Modeling*, ACM Tr. on CPS, Vol. 1, No. 1, November 2016

Fundamental Limits of Cyber-Physical Systems Modeling

EDWARD A. LEE, EECS Department, UC Berkeley

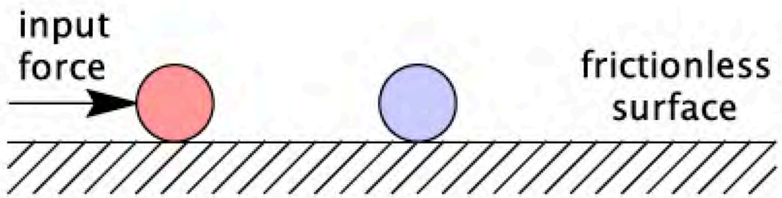
This article examines the role of modeling in the engineering of cyber-physical systems. It argues that the role that models play in engineering is different from the role they play in science, and that this difference should direct us to use a different class of models, where simplicity and clarity of semantics dominate over accuracy and detail. I argue that determinism in models used for engineering is a valuable property and should be preserved whenever possible, regardless of whether the system being modeled is deterministic. I then identify three classes of fundamental limits on modeling, specifically chaotic behavior, the inability of computers to numerically handle a continuum, and the incompleteness of determinism. The last of these has profound consequences.

CCS Concepts: • **Theory of computation** → **Timed and hybrid models**; • **Computing methodologies** → *Modeling methodologies*; • **Software and its engineering** → Domain specific languages

Additional Key Words and Phrases: Chaos, continuums, completeness

ACM Reference Format:
Edward A. Lee. 2016. Fundamental limits of cyber-physical systems modeling. ACM Trans. Cyber-Phys. Syst. 1, 1, Article 3 (November 2016), 26 pages.
DOI: http://dx.doi.org/10.1145/2912149

Illustration of the Incompleteness of Determinism



Conservation of momentum:

$$m_1 v_1' + m_2 v_2' = m_1 v_1 + m_2 v_2.$$

Conservation of kinetic energy:

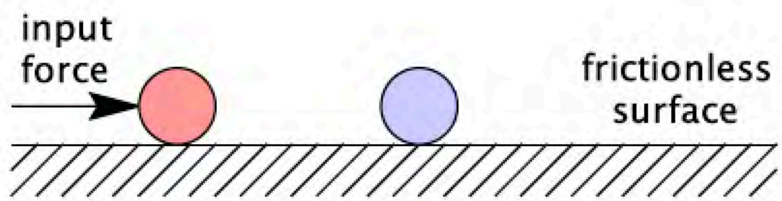
$$\frac{m_1 (v_1')^2}{2} + \frac{m_2 (v_2')^2}{2} = \frac{m_1 (v_1)^2}{2} + \frac{m_2 (v_2)^2}{2}.$$

We have two equations and two unknowns, v_1' and v_2'.

Illustration of the Incompleteness of Determinism

Quadratic problem has two solutions.

Solution 1: $v_1' = v_1$, $v_2' = v_2$
(ignore collision).



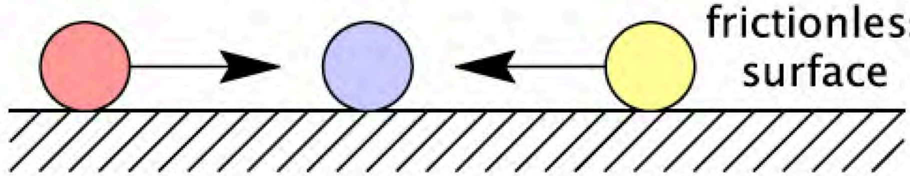
Solution 2:

$$\begin{aligned} v_1' &= \frac{v_1(m_1 - m_2) + 2m_2 v_2}{m_1 + m_2} \\ v_2' &= \frac{v_2(m_2 - m_1) + 2m_1 v_1}{m_1 + m_2}. \end{aligned}$$

Note that if $m_1 = m_2$, then the two masses simply exchange velocities (Newton's cradle).

Illustration of the Incompleteness of Determinism

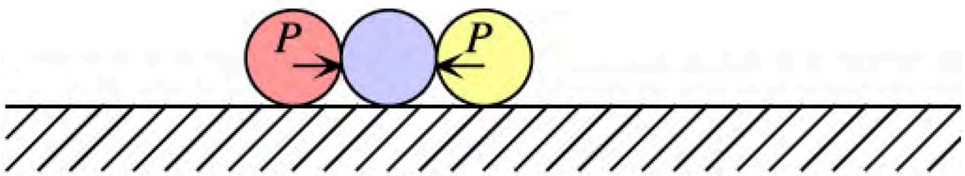
Consider this scenario:



Simultaneous collisions where one collision does not cause the other.

Illustration of the Incompleteness of Determinism

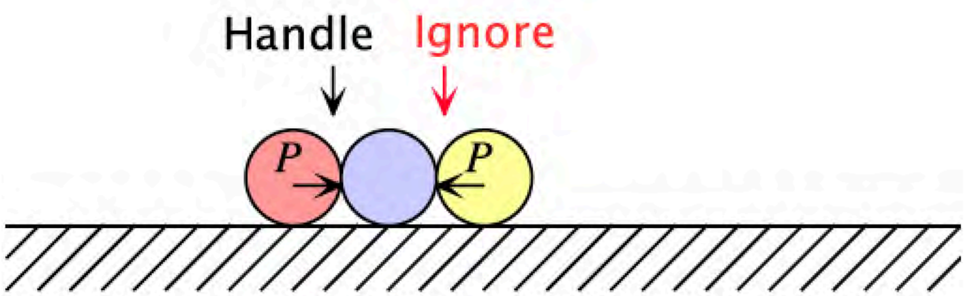
One solution: nondeterministic interleaving of the collisions:



At superdense time $(\tau, 0)$, we have two simultaneous collisions.

Illustration of the Incompleteness of Determinism

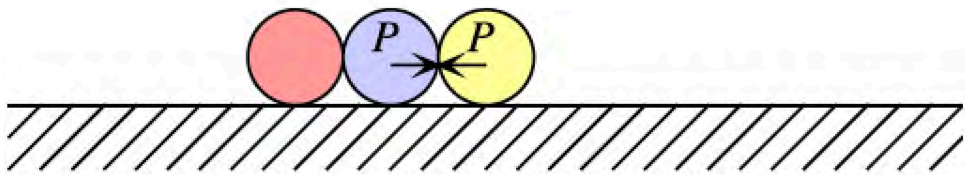
One solution: nondeterministic interleaving of the collisions:



At superdense time $(\tau, 1)$, choose arbitrarily to handle the left collision.

Illustration of the Incompleteness of Determinism

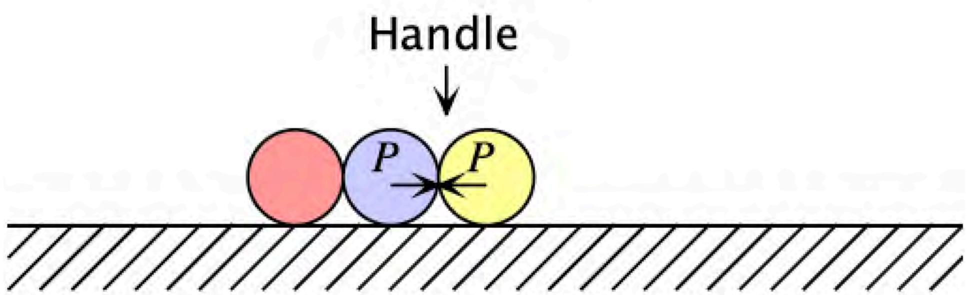
One solution: nondeterministic interleaving of the collisions:



After superdense time $(\tau, 1)$, the momentums are as shown.

Illustration of the Incompleteness of Determinism

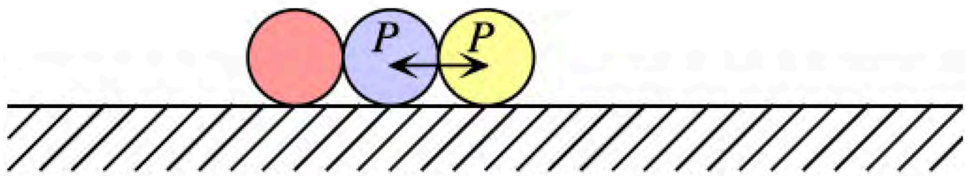
One solution: nondeterministic interleaving of the collisions:



At superdense time $(\tau, 2)$, handle the new collision.

Illustration of the Incompleteness of Determinism

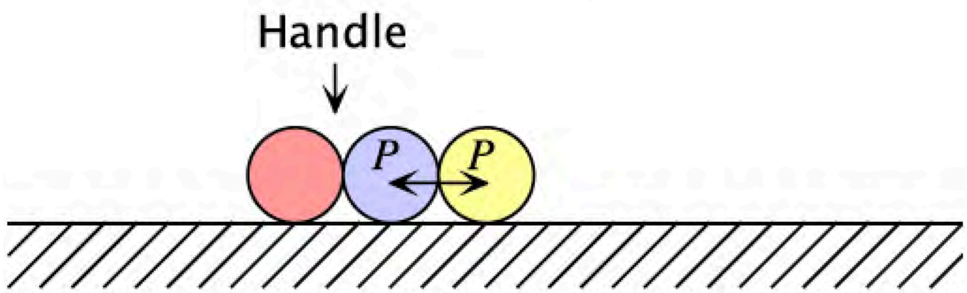
One solution: nondeterministic interleaving of the collisions:



After superdense time $(\tau, 2)$, the momentums are as shown.

Illustration of the Incompleteness of Determinism

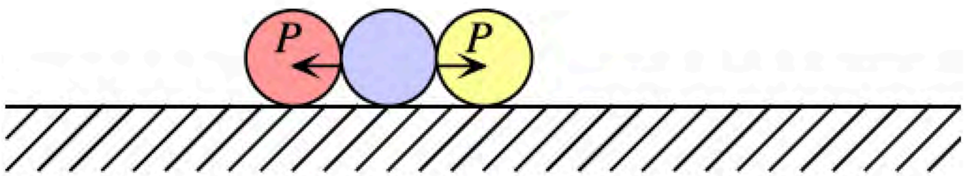
One solution: nondeterministic interleaving of the collisions:



At superdense time $(\tau, 3)$, handle the new collision.

Illustration of the Incompleteness of Determinism

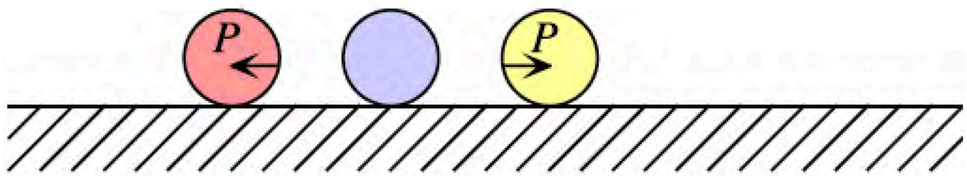
One solution: nondeterministic interleaving of the collisions:



After superdense time $(\tau, 3)$, the momentums are as shown.

Illustration of the Incompleteness of Determinism

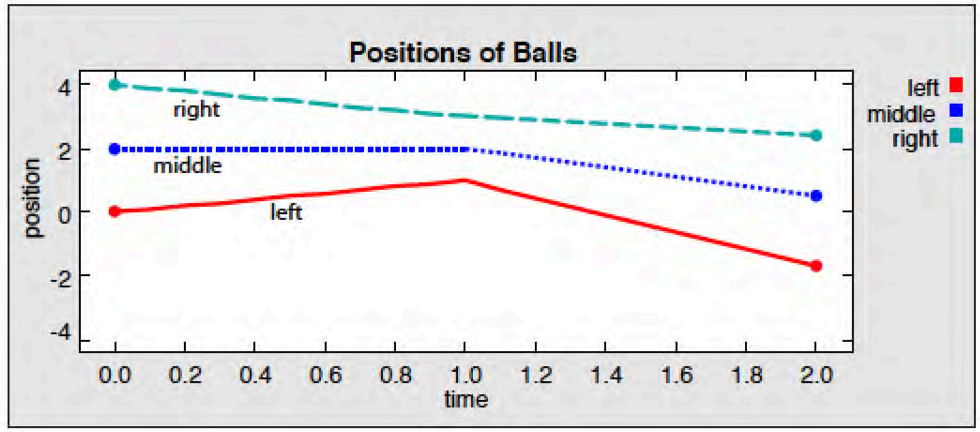
One solution: nondeterministic interleaving of the collisions:



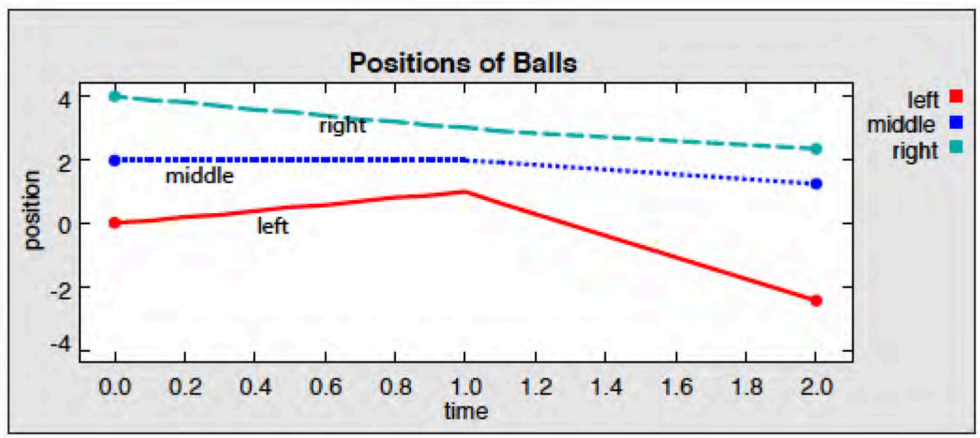
The balls move away at equal speed (if their masses are the same!)

Arbitrary Interleaving Yields Nondeterminism

If the masses are different, the behavior depends on which collision is handled first!



(a)

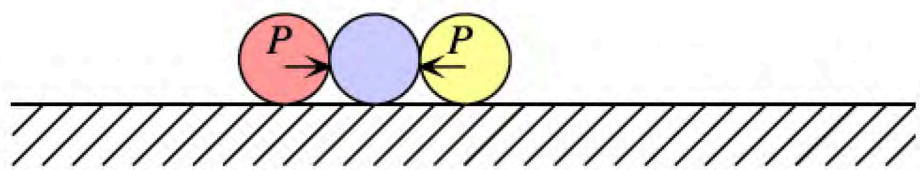


(b)

Recall the Heisenberg Uncertainty Principle

We cannot simultaneously know the position and momentum of an object to arbitrary precision.

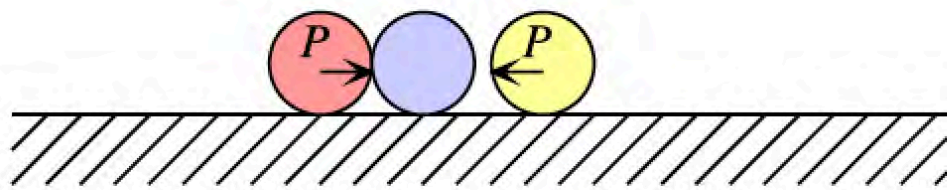
But the reaction to these collisions depends on knowing position and momentum precisely.



Is Determinism Incomplete?

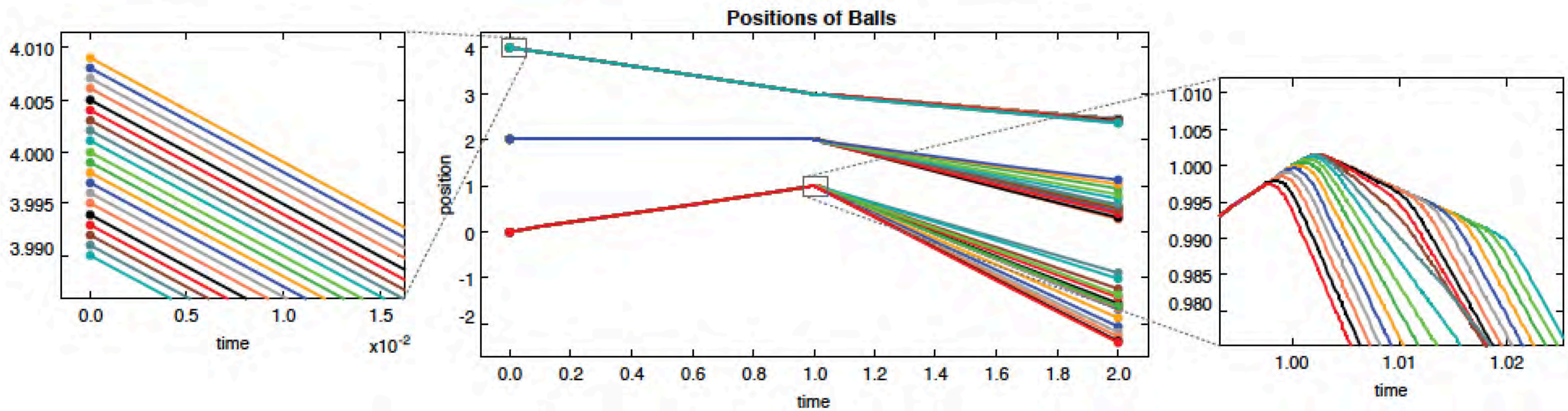
Let τ be the time between collisions. Consider a sequence of models for $\tau > 0$ where $\tau \to 0$.

Every model in the sequence is deterministic, but the limit model is not.



- In Lee (2017), I show that this sequence of models is Cauchy, so the space of deterministic models is incomplete (it does not contain its own limit points).
- In Lee (2014), I show that a direct description of this scenario results in a non-constructive model. The nondeterminism arises in making this model constructive.

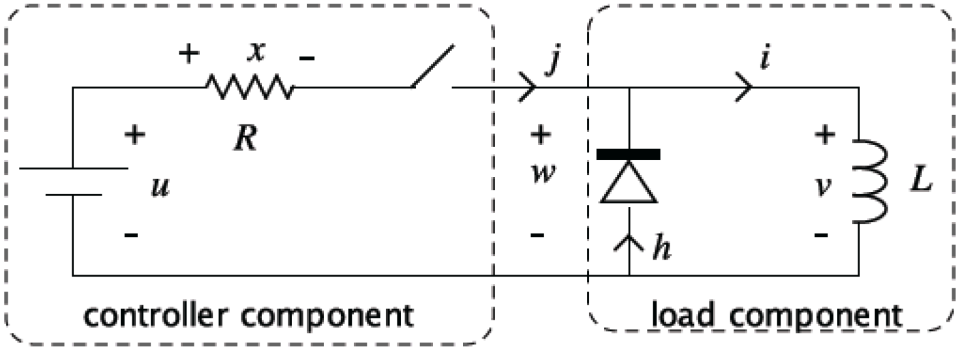
Rejecting discreteness leads to deterministic chaos



A continuous deterministic model that models the balls as springs is chaotic.

Discrete behaviors cannot be excluded unless we also reject causality

Example from Lee,
"Constructive Models of Discrete and Continuous Physical Phenomena,"
IEEE Access, 2014



A flyback diode is a commonly used circuit that prevents arcing when disconnecting an inductive load (like a motor) from a power source.

When the switch goes from closed to open, the causality and direct feedthrough properties of the two components reverse.

There is no logic that can transition from A causes B to B causes A smoothly without passing through non-constructive models.

Summary

- Deterministic models are extremely useful.
- Combining of our best deterministic cyber models and physical models today yields nondeterministic models.
- But deterministic models with faithful implementations exist (in research) for cyber-physical systems.
- Deterministic models aren't always possible or practical due to complexity, unknowns, chaos, and incompleteness.
- Determinism is a powerful modeling tool.
  Use it if you can. Back off only when you can't.

Conclusion

Models play a central role in reasoning about and designing engineered systems.

Determinism is a valuable and subtle property of models.

Plato and the Nerd
On Technology and Creativity

Edward Ashford Lee
MIT Press, 2017

Forthcoming book
My first for a general audience