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Carolin Martin, Noemi Schmitt and Frank Westerhoff

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Bamberg Economic Research Group
Bamberg University
Feldkirchenstraße 21
D-96052 Bamberg
Telefax: (0951) 863 5547
Telephone: (0951) 863 2687
felix.stuebben@uni-bamberg.de
http://www.uni-bamberg.de/vwl/forschung/berg/

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Redaktion:
Dr. Felix Stübben*

* felix.stuebben@uni-bamberg.de

Heterogeneous expectations, housing bubbles and tax policy

Carolin Martin, Noemi Schmitt and Frank Westerhoff*

University of Bamberg, Department of Economics,
Feldkirchenstrasse 21, 96045 Bamberg, Germany

Abstract

We integrate a plausible expectation formation and learning scheme of boundedly rational investors into a standard user cost housing market model, involving a rental and a housing capital market. In particular, investors switch between heterogeneous expectation rules according to an evolutionary fitness measure, given by the rules' past profitability. We analytically show that our housing market model may produce endogenous boom-bust dynamics. Furthermore, we demonstrate that policy makers may use our model as a tool to explore how different tax policies may affect the housing market's steady state, its stability and out-of-equilibrium behavior.

Keywords: Housing markets, bubbles and crashes, heterogeneous expectations, bounded rationality and learning, tax policy, steady state and stability analysis
JEL classification: D84, H24, R31

1. Introduction

Glaeser (2013), Shiller (2015) and Piazzesi and Schneider (2016) stress the fact that history is replete with dramatic housing market bubbles that had serious effects on the real economy. Unfortunately, the reasons for such market turbulence are still not well understood. Against this backdrop, the goal of our paper is twofold. First, we propose a novel model to enhance our understanding of the complex boom-and-bust behavior of housing markets. Second, we use our model to explore the extent to which policy makers can influence such dynamics by adjusting housing market-related taxes.

*Corresponding author's email address: frank.westerhoff@uni-bamberg.de

Our model reveals that endogenous housing market fluctuations may emerge through the interaction of real and behavioral forces. The real forces acting inside our model originate from a standard user cost housing market setup in the spirit of Poterba (1984, 1991), involving a rental and a housing capital market; these forces tie basic relations between house prices, the housing stock and the rent level. The model's behavioral forces are due to the expectation formation behavior of housing market investors who display a boundedly rational learning behavior, as put forward by Brock and Hommes (1997, 1998). Accordingly, investors choose between extrapolative and regressive expectation rules to forecast the future evolution of housing markets, based on the rules' relative past profitability – an assumption that is also in line with empirical observations (Case and Shiller 2003, Hommes 2011, Case et al. 2013, Bao and Hommes 2019). With a view to the omnipresent wilderness-of-bounded-rationality critique, Glaeser (2013) and Hommes (2013) argue that a simple and plausible rule-governed expectation formation scheme describes reality better than a framework with fully rational expectations. Obviously, we follow their line of reasoning in our paper.

Despite the behavioral nature of our model, it possesses a unique (fundamental) steady state, given by the discounted value of future risk-adjusted rents or, in the jargon of the housing market literature, by the relation between risk-adjusted rents and the user cost of housing. Moreover, we analytically derive the conditions under which the housing market's steady state becomes unstable. As it turns out, the steady state's stability domain depends on the housing market's real and behavioral side. For instance, higher interest rates are beneficial for market stability, while the housing market loses its stability and starts to display significant oscillatory fluctuations if investors rely heavily on the extrapolative expectation rule. Policy makers may therefore seek to stabilize the housing market by imposing housing market-related taxes. Using a mix of analytical and numerical tools, our model allows policy makers to clarify how such taxes may affect the housing market's steady state, its stability and out-of-equilibrium behavior.

Our paper is organized as follows. After reviewing some related literature in Section 2, we present our model in Section 3. In Section 4, we study its steady state, stability and out-of-equilibrium behavior, and explore how different tax regimes affect the main properties of our model in Section 5. In Section 6, we conclude our paper.

2. Related literature

To embed our contribution into the literature, let us briefly discuss some related research in which the effects of expectations and taxes on housing market dynamics were studied. First of all, it is important to note that Poterba (1984) developed his user cost model to explore the extent to which the U.S. housing market boom in the 1970s can be explained by changes in housing market-related taxes. In particular, he demonstrates that a decrease in property taxes led to a reduction of the user cost of housing, which, in turn, was at least partially responsible for the substantial house price increase at that time. Poterba (1984) assumes that housing market investors have perfect foresight, implying that his model exhibits the saddle-path stability property. If the steady state of his model is disturbed, there is a unique path (the so-called "stable arm") along which the system will approach its new steady state. The adjustment path can be summarized as follows. At the time of the shock, the house price overshoots its new steady state since the housing stock is initially fixed. As the housing stock also begins to adjust towards its new steady-state value, the house price monotonically converges towards its new steady state.

While Poterba (1991) underscores the fact that changing tax policies were an important contributory factor in the house price rise in the late 1970s, he also admits that his user cost argument is less able to explain the consequent housing market decline. To better understand the boom-bust behavior of housing markets, Poterba (1991) recommends to take into account the possibility of housing market investors extrapolating past price changes. Weil (1991) and Shiller (1991) strongly agree with this view. For instance, Weil (1991, p. 188) states that *"economists are going to have to bite the bullet and look at models that allow for not-fully rational expectations"*, advocating, amongst others, the modeling of extrapolative expectations. Moreover, Shiller (1991, p. 189) questions both the efficiency of housing markets and investors' forecasting ability, and ultimately stresses that there *"appears to be a purely speculative component of real estate prices"*. In the end, Poterba (1992) concludes that his user cost framework allows a clear-cut analysis of how tax reforms affect the steady-state levels of house prices, the rent level and the stock of housing, but that future work needs to study the dynamics and adjustment processes of (inefficient) housing markets in more detail.[1]

Despite such prominent encouragements, an empirically motivated modeling of market participants' house price expectations within a dynamic context is only slowly taking place in the economic profession. A rare and early exception is Wheaton (1999), who demonstrates that Poterba's (1984, 1991) user cost framework can produce more realistic oscillatory house price dynamics if housing market investors follow a simple rule-of-thumb behavior to predict house prices. Furthermore, Dieci and Westerhoff (2012) present a model of a speculative housing market in which housing market investors switch between extrapolative and regressive expectations, subject to the market's mispricing, thereby generating complex house price fluctuations. In the same vein, Kouwenberg and Zwinkels (2014), Eichholtz et al. (2015), Diks and Wang (2016), Bolt et al. (2019), Bao and Hommes (2019) and ter Ellen et al. (2020) show that models with extrapolators and mean-reversion believers may help us to explain the boom-bust behavior of housing markets.

Another line of research is motivated by Shiller's (2015) observation that mass psychology and investor sentiment are elements that play an important role in the determination of house prices. Burnside et al. (2016) explain the irregular boom-bust behavior of housing markets by a model in which housing market investors' projection of the future (fundamental) state of the housing market is either optimistic or pessimistic; they show that irregular boom-bust house price dynamics may occur due to waves of optimism and pessimism. Piazzesi and Schneider (2009) show that even a small fraction of optimistic housing market investors may be enough to trigger a housing market bubble. Glaeser and Nathanson (2017) propose a powerful framework in which housing market investors are boundedly rational, overconfident and extrapolate past house price changes into the future. Interestingly, their calibrated model matches key stylized facts of housing markets quite well, thereby underscoring the explanatory power of models that deviate from the assumption of full rationality.

However, our work is related more closely to the papers by Dieci and Westerhoff

[1] Further tax-related housing market papers with a similar spirit include Poterba and Sinai (2008) and Himmelberg et al. (2005). Poterba's (1984, 1991) model has been extended in many more directions. See Glaeser and Nathanson (2015) for a recent appraisal.

(2016) and Schmitt and Westerhoff (2019). Dieci and Westerhoff (2016) present a discrete-time generalization of Poterba's (1984, 1991) user cost model in which housing market investors can choose between different expectation rules, subject to market circumstances. Their goal is to explore how the housing market's supply side, in connection with speculative forces, may trigger and shape boom-bust dynamics. Schmitt and Westerhoff (2019) assume that risk-neutral housing market investors switch between extrapolative and regressive expectation rules with respect to the rules' forecasting accuracy. They show that endogenous housing market dynamics, characterized by short-run momentum, long-run mean reversion and excess volatility, may only arise if investors rely heavily on extrapolative expectations. Note that this is one of the few contributions in the field where market participants display a boundedly rational learning behavior – an important model ingredient to counter the wilderness-of-bounded-rationality criticism, as advocated in Glaeser (2013) and Hommes (2013).

In this paper, we follow Brock and Hommes (1997, 1998) by assuming that housing market investors are risk averse and switch between competing expectation rules, subject to the rules' past profitability.[2] Despite investors' learning behavior, our model may produce endogenous boom-bust housing market dynamics. In addition, we demonstrate that our model may serve as a framework to explore the extent to which policy makers may stabilize housing markets by adjusting the tax code. As we will see, policy makers have the opportunity to affect the housing market via the housing market's real and behavioral side. As far as we are aware, such a modeling and policy perspective is new in this line of research.

3. The basic model framework

Our model combines the housing market framework by Poterba (1984, 1991) with the heuristic switching approach by Brock and Hommes (1997, 1998). In particular, the housing market consists of two interrelated markets – a rental market and a housing capital market – that fix basic relations between house prices, the housing stock and the

[2] The heuristic switching approach by Brock and Hommes (1997, 1998) has been used in numerous models and applications. Powerful examples include Droste et al. (2002), de Grauwe and Grimaldi (2006), Boswijk et al. (2007) and Anufriev and Hommes (2012). Dieci and He (2018) provide a detailed review of this field, and also discuss its connection with the housing market literature.

rent level. Investors' demand for housing stock depends on their house price expectations. Motivated by the aforementioned theoretical and empirical literature, we assume that housing market investors select between an extrapolative and a regressive expectation rule to forecast future house prices, depending on the evolutionary fitness of these rules, measured in terms of past realized profits. For ease of exposition, we first abstain from considering housing market-related taxes. These will be introduced in Section 5.

Let us turn to the details of the model. Market clearing in the rental market takes place in every period t, implying that the demand for housing services D_t is equal to the supply of housing services S_t, i.e.

$$D_t = S_t. \tag{1}$$

Demand for housing services is assumed to be linearly decreasing at the current rent level R_t, the price of housing services, and is formalized as

$$D_t = a - bR_t, \tag{2}$$

where a and b are positive parameters. The supply of housing services is proportional to the current housing stock H_t, and can be expressed by

$$S_t = cH_t, \tag{3}$$

where $c > 0$. Combining (1), (2) and (3) reveals that the rent level R_t depends negatively on the existing housing stock, i.e.

$$R_t = \alpha - \beta H_t, \tag{4}$$

where $\alpha = \frac{a}{b} > 0$ is a scaling parameter and $\beta = \frac{c}{b} > 0$ denotes the sensitivity of the rent level with respect to the housing stock.[3] Of course, the model parameters have to ensure that $R_t > 0$.

Housing market investors can invest in a risk-free asset or in housing capital over the time horizon from period t to period $t+1$. The risk-free asset pays a fixed rate of return $r > 0$, while housing generates (imputed) rents R_t, which are fixed at the beginning of the period. By defining P_t as a hypothetical house price level at time t, the wealth of

[3] Proposition 1 reveals that parameter α affects the level of the model's steady state, but not its stability domain. For this reason, α may be regarded as a scaling parameter.

investor i in period $t+1$ is given by

$$W_{t+1}^i = (1+r)W_t^i + Z_t^i(P_{t+1} + R_t - (1+r+\delta)P_t) - c^i, \qquad (5)$$

where W_t^i and Z_t^i represent the wealth and demand for housing stock of investor i at time t. Note that parameter $0 < \delta < 1$ denotes the housing depreciation rate, and parameter $c^i \geq 0$ captures possible costs associated with investors i's investment behavior. Moreover, variables indexed with $t+1$ are random.

Housing market investors are assumed to be myopic mean-variance maximizers, implying that their demand for housing stock follows from

$$max_{Z_t^i} \left\{ E_t^i[W_{t+1}^i] - \frac{\lambda^i}{2} V_t^i[W_{t+1}^i] \right\}, \qquad (6)$$

where $E_t^i[W_{t+1}^i]$ and $V_t^i[W_{t+1}^i]$ describe the belief of investor i about the conditional expectation and conditional variance of his wealth in period $t+1$, while parameter λ^i denotes the corresponding (absolute) risk aversion. Solving (6) for Z_t^i then yields

$$Z_t^i = \frac{E_t^i[P_{t+1}] + R_t - (1+r+\delta)P_t}{\lambda^i V_t^i[P_{t+1}]}. \qquad (7)$$

Accordingly, investor i's optimal demand for housing stock depends positively on house price expectations and the rent level, and negatively on the interest rate, the depreciation rate, the current house price and the perceived housing market risk.

In the following, we introduce a few simplifying assumptions. First, investors' beliefs about conditional variance of the price are constant for all t and uniform across all investors i, i.e. $V_t^i[P_{t+1}] = \sigma^2 > 0$. Second, all investors have the same risk aversion, i.e. $\lambda^i = \lambda > 0$. Therefore, investors' total housing demand can be expressed as $Z_t = \sum_{i=1}^{N} Z_t^i = \frac{N}{\lambda\sigma^2}\left(\frac{1}{N}\sum_{i=1}^{N} E_t^i[P_{t+1}] + R_t - (1+r+\delta)P_t\right)$. Finally, by denoting investors' average house price expectations by $E_t[P_{t+1}] = \frac{1}{N}\sum_{i=1}^{N} E_t^i[P_{t+1}]$ and normalizing the mass of investors to $N=1$, we obtain

$$Z_t = \frac{E_t[P_{t+1}] + R_t - (1+r+\delta)P_t}{\lambda\sigma^2}. \qquad (8)$$

As equilibrium of demand and supply in the housing capital market is given by

$$Z_t = H_t, \qquad (9)$$

the market clearing price P_t can be expressed as

$$P_t = \frac{E_t[P_{t+1}] + \hat{R}_t}{1+r+\delta}, \qquad (10)$$

where $\hat{R}_t = R_t - H_t \lambda \sigma^2$. Accordingly, the house price is equal to the discounted value of investors' next period's average house price expectation plus risk-adjusted rent payments; a standard no-arbitrage condition common in models with an asset-pricing nature.

The housing stock evolves as

$$H_t = I_t + (1 - \delta) H_{t-1}, \tag{11}$$

where I_t indicates the amount of new housing construction in period t. Note that we assume that houses are built with a one-period production lag. Moreover, home builders are risk neutral, and maximize expected profits, subject to a quadratic cost function, i.e. $max_{I_t}\{E_{t-1}[P_t]I_t - C_t\}$, where $C_t = \frac{1}{2\gamma} I_t^2$. Consequently, new housing construction is given by

$$I_t = \gamma E_{t-1}[P_t], \tag{12}$$

where $\gamma > 0$ is an inverse cost parameter which implies that a lower value of γ generates higher building costs and a more sluggish housing supply. By assuming that home builders form naive expectations, i.e. $E_{t-1}[P_t] = P_{t-1}$, the evolution of the housing stock can be rewritten as

$$H_t = \gamma P_{t-1} + (1 - \delta) H_{t-1}. \tag{13}$$

Let us now turn to the expectation formation behavior of housing market investors. Inspired by Brock and Hommes (1997, 1998), investors select between competing expectation rules to forecast future house prices. In this paper, we concentrate on two representative types of expectation rules: a free extrapolative expectation rule, denoted by $E_t^E[P_{t+1}]$, and a costly regressive expectation rule, i.e. $E_t^R[P_{t+1}]$. Investors' average house price expectations can thus be defined as

$$E_t[P_{t+1}] = N_t^E E_t^E[P_{t+1}] + N_t^R E_t^R[P_{t+1}], \tag{14}$$

where N_t^E and N_t^R stand for the market shares of investors relying on extrapolative and regressive expectations, respectively. Extrapolative expectations presume that house prices move in trends; it can be expressed by

$$E_t^E[P_{t+1}] = P_{t-1} + \chi(P_{t-1} - P_{t-2}). \tag{15}$$

Accordingly, extrapolators pay attention to the most recent price trend, where $\chi \geq 0$ indicates how strongly investors extrapolate past house price trends into the future.

For $\chi = 0$, (15) implies naive expectations. In contrast, regressive expectations are formalized as

$$E_t^R[P_{t+1}] = P_{t-1} + \phi(F - P_{t-1}), \tag{16}$$

where F represents the housing market's fundamental value and $0 < \phi \leq 1$ the expected adjustment speed. Thus, investors who follow this rule believe that house prices will return towards their fundamental value over time.[4] Note that both expectation rules forecast in period t the house price for period $t+1$, conditional on the information set available at period $t-1$.

In each time step, housing market investors have to determine which expectation rule to follow. This decision depends on the rules' fitness. We assume that the higher the fitness of an expectation rule, the more investors will follow it. As in Brock and Hommes (1997, 1998), and based on Manski and McFadden (1981), we update the market share of investors using the extrapolative and the regressive expectation rule via the multinomial discrete-choice model. Therefore, we obtain

$$N_t^E = \frac{\exp[\nu A_t^E]}{\exp[\nu A_t^E] + \exp[\nu A_t^R]} \tag{17}$$

and

$$N_t^R = \frac{\exp[\nu A_t^R]}{\exp[\nu A_t^E] + \exp[\nu A_t^R]}, \tag{18}$$

where A_t^E and A_t^R denote the fitness of extrapolative and regressive expectations in period t, respectively. Parameter $\nu \geq 0$ measures how sensitively investors choose the most attractive expectation rule. For $\nu = 0$, investors do not observe any fitness differentials between the two expectations rules, and both market shares will be equal to $\frac{1}{2}$. As the intensity of choice parameter ν increases, more and more investors switch to the expectation rule with the higher fitness. For $\nu \to +\infty$, fitness differentials are perfectly observed, and all investors opt for the expectation rule yielding the highest fitness. Since the weights of the two expectation rules add up to 1, the market share of extrapolative (regressive) expectations can also be written as $N_t^E = 1 - N_t^R$ ($N_t^R = 1 - N_t^E$).

The fitness of the two expectation rules in period t depends on realized past profits

[4] An implicit assumption we make is that investors make no prediction errors at the steady state. As shown in the Appendix, F therefore corresponds to the housing markets' unique steady-state price, reflecting the discounted value of future risk-adjusted rent payments.

and can be described by

$$A_t^E = (P_{t-1} + R_{t-2} - (1 + r + \delta)P_{t-2})Z_{t-2}^E \tag{19}$$

and

$$A_t^R = (P_{t-1} + R_{t-2} - (1 + r + \delta)P_{t-2})Z_{t-2}^R - c, \tag{20}$$

where

$$Z_t^E = \frac{E_t^E[P_{t+1}] + R_t - (1 + r + \delta)P_t}{\lambda\sigma^2} \tag{21}$$

and

$$Z_t^R = \frac{E_t^R[P_{t+1}] + R_t - (1 + r + \delta)P_t}{\lambda\sigma^2} \tag{22}$$

represent investors' demand for housing stock in period t when forming extrapolative and regressive expectations, respectively. Note that it may be costly to use the regressive expectation rule since investors have to acquire some kind of knowledge about the economy. In particular, investors have to examine what the housing market's fundamental house price will be, and how quickly the housing market will return to this value. This effort is captured by the information cost parameter $c \geq 0$, and reduces the fitness of regressive expectations. As pointed out by Hommes (2013), realized net profits are a natural candidate for an evolutionary fitness measure since this is what investors seem to care about most in real markets.

4. Implications of our basic model framework

We now explore our basic model framework. In Section 4.1, we first present our main analytical results. In Section 4.2, we then continue with a numerical investigation of our model.

4.1. Analytical insights

In the Appendix, we show that the dynamics of our model is driven by a six-dimensional nonlinear map, and prove the following results.

Proposition 1. *The model's unique steady state, implying, amongst others, that* $\overline{P} = F = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)\gamma} = \frac{\overline{R}-\lambda\sigma^2\overline{H}}{r+\delta}$, $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *loses its local asymptotic stability if either*

$$(i)\ \ \overline{N}^E\chi\delta + \frac{\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} \ < \ \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$$

or

$$(ii)\ \ \overline{N}^R\phi + \frac{\gamma(\beta+\lambda\sigma^2)}{2-\delta} \ < \ 2+r+\delta+2\overline{N}^E\chi$$

becomes violated, where $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[\nu c]}$, *respectively. Moreover, a violation of the first (second) inequality is associated with a Neimark-Sacker (Flip) bifurcation.*

Proposition 1 deserves comment. Let us start with the properties of the model's steady state. Note that $\overline{P}$, $\overline{H}$ and $\overline{R}$ are independent of any behavioral parameters. Since $\overline{P} = F = \frac{\overline{R}-\lambda\sigma^2\overline{H}}{r+\delta} = \frac{\overline{\hat{R}}}{r+\delta}$, it becomes clear that investors discount future risk-adjusted rent payments to compute the housing market's fundamental value. This is also in line with Poterba (1984, 1991), who defines the term $r+\delta$ as the user cost of housing. Although he considers perfect foresight, our steady state is basically equivalent to the one in his models because our housing market investors make no prediction errors at the steady state. For this reason, we regard the model's unique steady state as a fundamental steady state.

Proposition 1 allows us to draw the following steady-state conclusions. An increase in the interest rate decreases investors' demand for housing stock and, consequently, leads to a reduction of the house price; a lower housing stock; and a higher rent level. Comparable effects are observed if housing market investors become more risk averse and/or perceive a higher housing market risk. If it gets cheaper to build new houses, i.e. if the inverse cost parameter γ increases, house prices as well as the rent level decrease, while the housing stock increases. A higher depreciation rate reduces the stock of housing and, consequently, yields a higher rent level. However, house prices only decrease if $\delta > \sqrt{(\beta+\lambda\sigma^2)\gamma}$. In this case, the effects of an increase in the interest rate and the depreciation rate are qualitatively the same. With respect to the parameters describing the rental market, we can conclude that an increase in the scaling parameter α increases the house price, the housing stock and the rent level, while an increase in the sensitivity parameter β causes the opposite. For completeness, we mention that $\overline{Z}^E = \overline{Z}^R = \overline{H}$, $\overline{A}^E = (\overline{R} - (r+\delta)\overline{P})\overline{H}$ and $\overline{A}^R = (\overline{R} - (r+\delta)\overline{P})\overline{H} - c$. Since $\overline{A}^E - \overline{A}^R = c$, the steady-state fractions of investors relying on extrapolative and regressive expectations, given by $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ and $\overline{N}^R = \frac{1}{1+exp[\nu c]}$, depend only on investors' intensity of choice and on the costs of forming regressive expectations.

Let us now turn to the steady state's stability properties. Note that both stability conditions depend on real and behavioral parameters. Since housing markets display cyclical dynamics, a phenomenon associated with a Neimark-Sacker bifurcation, our main focus is on Proposition 1's first stability condition.[5] First of all, if we assumed naive versus regressive expectations, i.e. $\chi = 0$ and $0 < \phi < 1$, the Neimark-Sacker condition would always be fulfilled. Cyclical housing market dynamics can thus only arise within our model if investors extrapolate past price changes. However, it is also obvious from stability condition (i) that cyclical housing market dynamics becomes less likely if investors expect house prices to return towards their fundamental value more quickly. Furthermore, the Neimark-Sacker condition is also violated when $\overline{N}^E\chi$ moves towards $1+r+\delta$ (see the denominator of the second term on the left-hand side). In this respect, it might be insightful to explore two extreme scenarios. If either information costs c or the intensity of choice parameter ν converge to infinity, then all investors form extrapolative expectations. Hence, stability will be lost at the latest as χ approaches $1+r+\delta$. If c and/or ν converge to zero, only half of investors form extrapolative expectations, and stability will be lost at the latest as χ approaches $2(1+r+\delta)$. In this sense, we can conclude that an increase in c or ν may destabilize the model's steady state. Finally, increasing values of the real parameters β, γ, λ and σ^2 harm the stability of housing markets, while an increase in r has a beneficial effect. Introducing the innocuous assumption that $r + \delta < 1$ furthermore reveals that an increase in the depreciation rate

[5] A Neimark-Sacker bifurcation occurs if the modulus of a pair of complex, conjugate eigenvalues crosses the unit circle, giving rise to periodic or quasi-periodic motion. The contributions by Wheaton (1999), Kouwenberg and Zwinkels (2014), Dieci and Westerhoff (2016), Glaeser and Nathanson (2017) and Bao and Hommes (2019) also focus on scenarios with complex, conjugate eigenvalues, seeking to explain the oscillatory boom-bust behavior of real housing markets, as documented by Glaeser (2013), Shiller (2015) and Piazzesi and Schneider (2016). In contrast, a Flip bifurcation requires that a real eigenvalue passes through -1, causing the emergence of a period-two cycle.

also contributes to the stability of housing markets.

From an economic perspective, the violation of the Flip bifurcation boundary, causing a period-two cycle, is of secondary importance. Nevertheless, the second stability condition of Proposition 1 reveals that an increase in parameter ϕ, capturing investors' expected mean reversion speed, may create a period-two cycle, provided that parameters β, γ, λ and σ^2 are sufficiently large. Such a bifurcation becomes more likely if the market share of regressive expectations increases, which is the case if information costs and/or investors' intensity of choice decrease. Finally, we note that an increase in the interest rate, the depreciation rate or investors' extrapolation strength may reverse a Flip bifurcation.

4.2. Numerical insights

Equipped with our analytical insights, we are now ready to explore the model's out-of-equilibrium behavior. Table 1 presents the base parameter setting for our numerical investigations. Since the interest rate and the depreciation rate are given by five percent, one time step in our simulations may roughly be regarded as one year. Accordingly, the production lag in the housing market is also given by about one year, which seems to be a reasonable choice for a model like ours. The remaining parameters are selected such that our model is able to mimic - at least in a qualitative sense - the boom-bust behavior of housing markets, as documented in Glaeser (2013), Shiller (2015) and Piazzesi and Schneider (2016). However, we remark that the behavioral parameters of our model, in particular those affecting investors' expectation formation, are in line with empirical and experimental observations (Case and Shiller 2003, Case et al. 2012, Anufriev and Hommes 2012, Bao and Hommes 2019 and ter Ellen et al. 2020).

Note that the base parameter setting implies that $\overline{P} = F = 1$, $\overline{H} = 20$, $\overline{R} = 0.3$ and $\overline{N}^E \approx 0.731$. Furthermore, the model's steady state is unstable. For instance, the extrapolation parameter, which is given by $\chi = 1.1$, is slightly above the Neimark-Sacker threshold $\chi^{NS}_{crit} \approx 1.08$ (while the Flip condition is not violated). And, in fact, the dynamics depicted in Figure 1 displays endogenous boom-bust housing market dynamics. To be precise, the panels show, from top to bottom, the evolution of house prices, the market share of extrapolators, the housing stock and the rent level, respectively. The simulation run comprises 30 observations; a longer transient period has been deleted.

Table 1: Base parameter setting

$\alpha = 2.3$	scaling parameter	rental market
$\beta = 0.1$	sensitivity of rental market	
$\gamma = 1$	sensitivity of home building	supply side of housing market
$\delta = 0.05$	depreciation rate	
$r = 0.05$	interest rate	
$\chi = 1.1$	extrapolative parameter	price expectations
$\phi = 0.6$	regressive parameter	
$\nu = 1$	intensity of choice	
$\lambda = 0.0025$	risk aversion	risk perception
$\sigma^2 = 4$	variance beliefs	
$c = 1$	information cost	fitness

The functioning of the model may be explained as follows. Suppose that the house price has just increased above its fundamental value. In such a situation, the extrapolative expectation rule has correctly predicted a further price increase, while regressive expectations have falsely predicted a reversion towards the fundamental value. For this reason, the extrapolative expectation rule is more profitable than the regressive one. As extrapolative expectations now attract more followers, house prices increase further. Eventually, however, the market loses momentum. This could happen for several reasons. First, the market share of extrapolators cannot grow forever. Second, the remaining investors who rely on the regressive expectation rule bet increasingly aggressively on a mean reversion of the housing market. Third, the housing stock has increased due to the construction of new housing during the formation of the bubble. This depresses the rent level and therefore dampens house prices, too. At the bubble's turning point, extrapolative expectations are wrong, while regressive expectations are right. But once the direction of the housing market reverses, both expectation rules correctly anticipate a downturn of the housing market. Moreover, new housing construction – due to house prices that are still relatively high – lets the housing stock grow for a few more periods, pushing the rent level down further. Together, these behavioral and real forces lead to an overshooting in the housing market, i.e. house prices drop below their fundamental value. Then, we once again have a situation in which extrapolative expectations produce more accurate predictions than the regressive expectation rule. However, investors' learning behavior depends on past realized profits, which is why the market share of extrapolators recovers with some delay. In between, the rent level increases again. Since

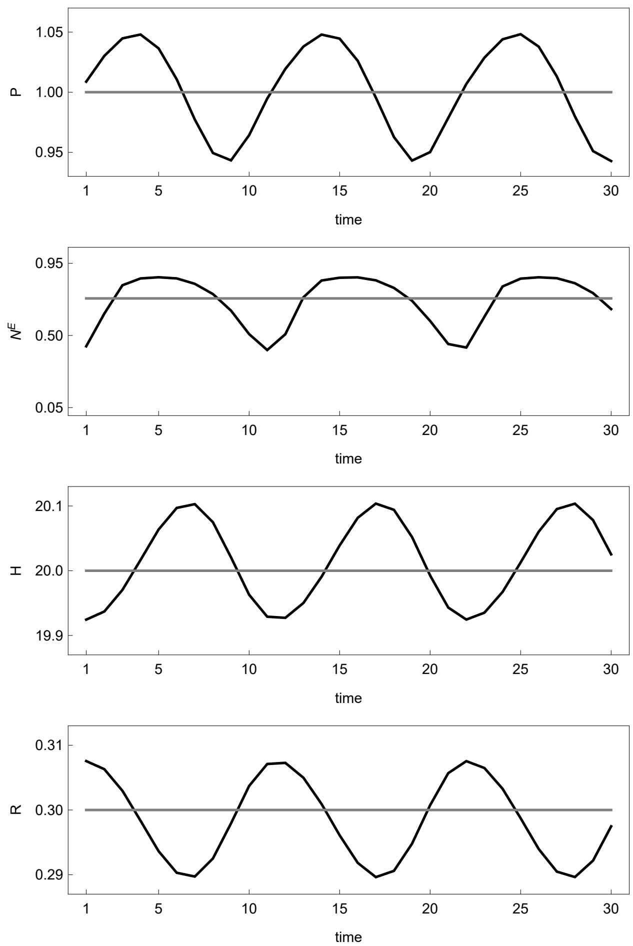

Figure 1: Snapshot of the model dynamics for our base parameter setting. The panels show, from top to bottom, the evolution of house prices, the market share of extrapolators, the housing stock and the rent level, respectively. The dynamics is depicted for 30 time steps; a longer transient period has been deleted.

a considerable fraction of investors still uses the regressive expectation rule, prices are pushed upwards, and we see the emergence of the next housing market bubble.

In fact, it is the complex interplay between real and behavioral forces that keeps the dynamics alive. While real forces, in particular the housing stock and rent adjustments, tend to stabilize the housing market, behavioral forces have a double-edged effect. Extrapolative expectations tend to push house prices away from fundamentals; regressive expectations, in turn, exercise mean-reversion pressure. Note that the boom-bust cycle depicted in Figure 1 repeats itself in a more or less regular manner. Figure 2 reveals,

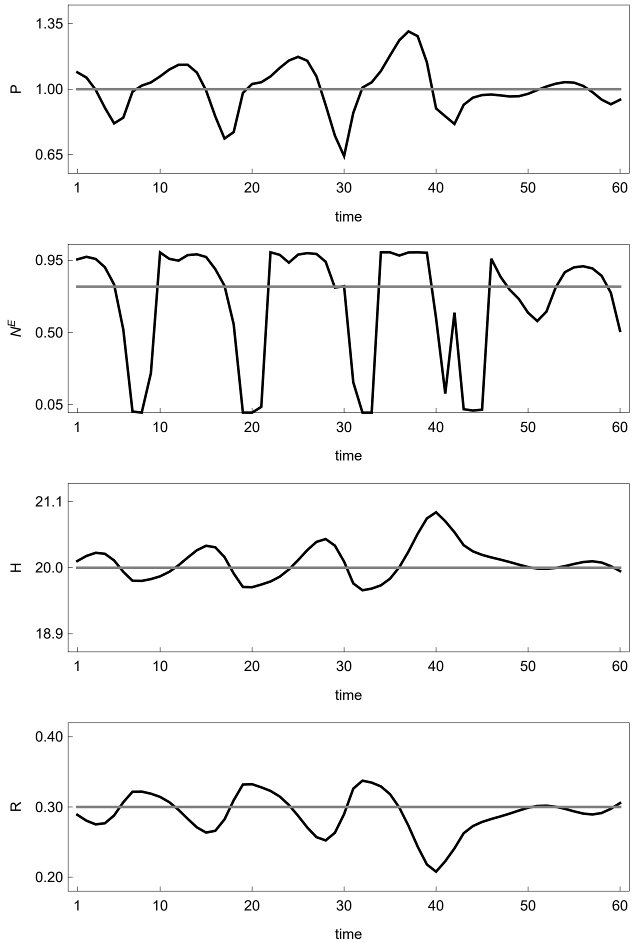

Figure 2: Snapshot of the model dynamics for an alternative parameter setting. The panels show, from top to bottom, the evolution of house prices, the market share of extrapolators, the housing stock and the rent level, respectively. The dynamics is depicted for 60 time steps; a longer transient period has been deleted. Parameter setting as in Figure 1, except that $\chi = 1.35$, $\phi = 0.75$ and $\nu = 1.3$.

however, that our model is also able to produce more irregular dynamics. The simulation run – now for 60 time steps – rests on the base parameter setting, except that $\chi = 1.35$, $\phi = 0.75$ and $\nu = 1.3$. These parameter changes leave the model's fundamental steady state unaffected, although $\overline{N}^E$ increases from 0.731 to 0.786. Of course, the model's instability is still due to a Neimark-Sacker bifurcation. As can be seen, stronger house price cycles result in strong housing stock oscillations, and thus in more volatile rent levels. Needless to say, irregular dynamics may also be observed in the presence of exogenous noise (not depicted), although our model may generate them completely endogenously.

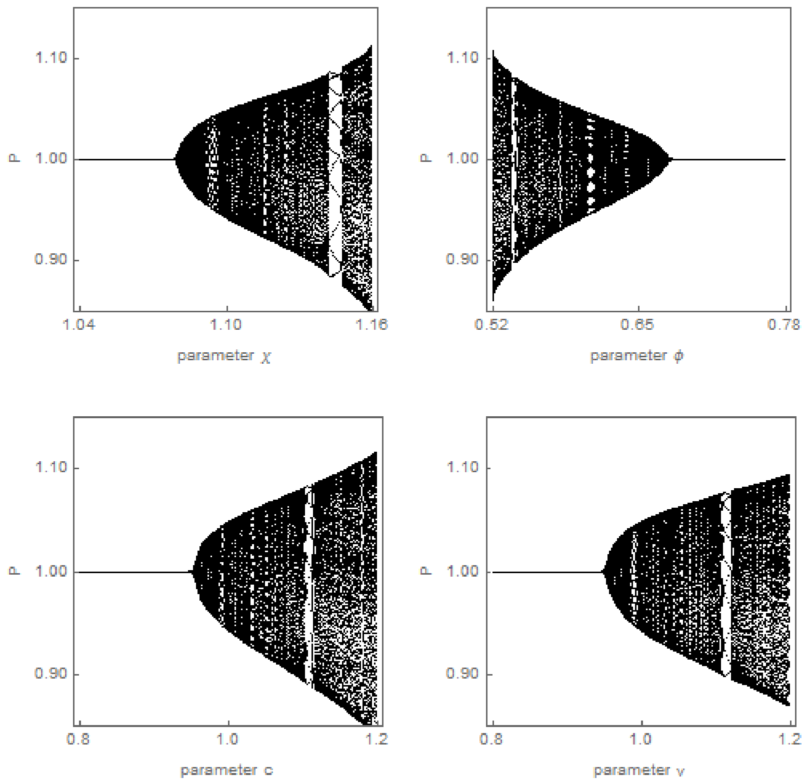

Figure 3: Effects of behavioral parameters on house price dynamics. The panels show, from top left to bottom right, bifurcation diagrams for the extrapolative parameter χ, the regressive parameter ϕ, information costs c and the intensity of choice ν. Parameters are as in our base parameter setting.

To demonstrate how the dynamics of our model depends on its parameters, we next present a number of bifurcation diagrams in Figure 3. Here, we provide examples of how our behavioral parameters χ, ϕ, c and ν may influence house price dynamics. The panels depict, from top left to bottom right, bifurcation diagrams for $1.04 < \chi < 1.16$, $0.52 < \phi < 0.78$, $0.8 < c < 1.2$ and $0.8 < \nu < 1.2$, respectively. As already stated in Proposition 1, these parameters do not affect the housing market's fundamental steady-state price, i.e. $\overline{P} = F = 1$. In the top left panel, the fundamental steady state is initially stable and loses its stability as soon as χ exceeds the critical value $\chi^{NS}_{crit} \approx 1.08$, for which endogenous quasi-periodic dynamics emerges. While the amplitude of house price fluctuations becomes larger if extrapolators react more aggressively to past house price trends, the top right panel shows that a stronger belief in mean reversion reduces their amplitude. In fact, a convergence to the steady state sets in when ϕ surpasses $\phi_{crit}^{NS} \approx 0.674$. Of course, these observations correspond to our analytical results, which are supported further by the bottom two panels. The bifurcation diagram for parameter c shows that the fundamental steady state becomes unstable at $c_{crit}^{NS} \approx 0.953$, after which the amplitude of house price dynamics increases with information costs. The reason for this is that rising information costs increasingly reduce the fitness of the stabilizing regressive expectation rule. Consequently, more and more investors switch to extrapolative expectations, which has a destabilizing impact on housing market dynamics. A very similar bifurcation route can be observed in the bottom right panel. As can be seen, at $\nu_{crit}^{NS} \approx 0.953$, the fixed-point dynamics turns into quasi-periodic motion. The destabilizing impact of an increasing intensity of choice can be easily explained. Recall that extrapolative expectations have a higher steady-state fitness than regressive expectations. Since investors react more sensitively to fitness differences as ν increases, more and more of them will opt for extrapolative expectations, which destabilizes the dynamics, and the amplitude of house price fluctuations increases.[6]

In Figure 4, we show how house prices react to an increase in the model's real parameters. The six panels show bifurcation diagrams for $0.02 < r < 0.1$, $0.045 < \delta < 0.065$, $0.9 < \gamma < 1.1$, $0 < \lambda < 0.004$, $0.09 < \beta < 0.11$ and $2.15 < \alpha < 2.45$. It can be seen from the top left panel that an increasing interest rate decreases the amplitude of house price fluctuations. Moreover, the quasi-periodic dynamics converges into a stable fixed point when r exceeds $r_{crit}^{NS} \approx 0.06$. However, the steady-state house price $\overline{P}$ decreases with r. Similar observations are apparent in the top right panel, where the amplitude of house price fluctuations becomes smaller when the depreciation rate increases. At $\delta_{crit}^{NS} \approx 0.058$, the quasi-periodic dynamics segues into our stable fundamental steady state which, in turn, increases with δ. The destabilizing impact of the inverse cost parameter γ, the risk aversion parameter λ and the rental market's sensitivity parameter β are presented in the two middle panels and the bottom left panel, respectively. Note that their bifurcation routes are very similar. For increasing

[6]Interestingly, similar effects of the behavioral parameters can be observed in the related asset-pricing and cobweb model by Brock and Hommes (1997, 1998).

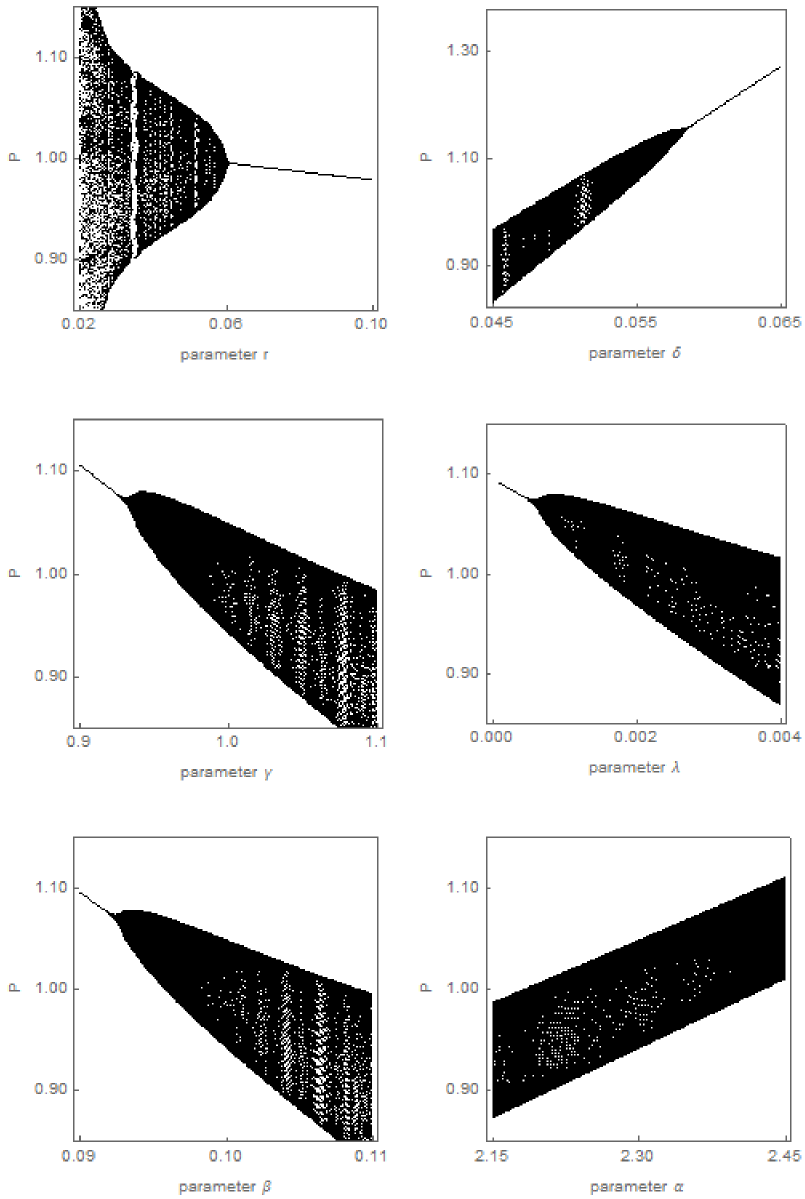

Figure 4: Effects of real parameters on house price dynamics. The panels show, from top left to bottom right, bifurcation diagrams for the interest rate r, the depreciation rate δ, the inverse cost parameter γ, the risk aversion parameter λ, the sensitivity parameter β and the scaling parameter α. Parameters are as in our base parameter setting.

values of parameters γ, λ and β, the stable steady-state house price decreases and becomes unstable at bifurcation values $\gamma_{crit}^{NS} \approx 0.933$, $\lambda_{crit}^{NS} \approx 0.00066$ and $\beta_{crit}^{NS} \approx 0.093$, respectively. At these points, a Neimark-Sacker bifurcation occurs, and the amplitude of house price fluctuations becomes larger as the parameters increase further. Finally, it becomes apparent from the bottom right panel that the scaling parameter α has no effect on the stability of housing market dynamics, as the amplitude of house price oscillations basically remains constant as α increases.

5. The housing market model with taxes

In Sections 5.1 to 5.6, we explore the model's steady state and stability properties as well as out-of-equilibrium effects when various different tax policies are considered. In doing so, we seek to demonstrate that our model provides a useful framework to address the effects of housing market-related taxes. With respect to the steady state's stability domain, we focus on the Neimark-Sacker stability condition. Our main results are summarized by Propositions 2 to 7 (their proofs are analogous to the one of Proposition 1, see the Appendix) and illustrated in bifurcation diagrams.

5.1. Tax on the value of houses

Let us start our analysis with the case of a property tax, imposed periodically (annually) on the value of houses. Note that such a tax affects investor i's wealth equation, which turns into

$$W_{t+1}^i = (1+r)W_t^i + Z_t^i(P_{t+1} + R_t - (1+r+\delta+\tau)P_t) - c^i, \tag{23}$$

where τ stands for the tax rate. Straightforward computations reveal that investors' total demand then becomes $Z_t = \frac{E_t[P_{t+1}]+R_t-(1+r+\delta+\tau)P_t}{\lambda\sigma^2}$, implying that house prices follow $P_t = \frac{E_t[P_{t+1}]+\hat{R}}{1+r+\delta+\tau}$. Moreover, the fitness functions of the extrapolative and regressive expectation rule now read $A_t^E = (P_{t-1} + R_{t-2} - (1+r+\delta+\tau)P_{t-2})Z_{t-2}^E$ and $A_t^R = (P_{t-1} + R_{t-2} - (1+r+\delta+\tau)P_{t-2})Z_{t-2}^R - c$, respectively. All other equations remain as before. Proposition 2 summarizes our main analytical results.

Proposition 2. *At the model's unique steady state, we have* $\overline{P} = \frac{\alpha\delta}{(r+\delta+\tau)\delta+(\beta+\lambda\sigma^2)\gamma} = F$, $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *implying that* $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[\nu c]}$. *Suppose that the steady state is locally asymptotically stable. If* $\overline{N}^E\chi\delta + \frac{\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta+\tau-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r+\tau}{1-\delta}$ *is violated, a Neimark-Sacker bifurcation occurs.*

A comparison of Propositions 1 and 2 shows that an increase in the property tax rate τ has quite similar effects as an increase in the interest rate r. More precisely, higher tax rates make the housing market less attractive for investors. Therefore, the demand for housing stock decreases, which causes the fundamental house price $\overline{P}$ to fall. Consequently, the fundamental housing stock $\overline{H}$ declines and the fundamental rent level $\overline{R}$ increases. Importantly, higher tax rates may prevent a Neimark-Sacker bifurcation. Note that this also becomes obvious from the bifurcation diagram depicted in the top left panel of Figure 5. Here, we use our base parameter setting, except that τ is varied between 0 and 0.1, and show how the dynamics of the housing market depends on the tax rate. As can be seen, higher property taxes rates initially decrease the amplitude of house price fluctuations, and a convergence to the steady state sets in when τ exceeds $\tau_{crit}^{NS} \approx 0.010$. We can therefore conclude that a property tax has a stabilizing effect on housing markets, although it also yields lower house prices and, consequently, a lower housing stock and higher rent levels.

5.2. Tax on rental income

A tax on rental income changes investor i's wealth equation to

$$W_{t+1}^i = (1+r)W_t^i + Z_t^i(P_{t+1} + (1-\tau)R_t - (1+r+\delta)P_t) - c^i, \tag{24}$$

where τ again denotes the tax rate imposed by policy makers. Accordingly, investors' total demand becomes $Z_t = \frac{E_t[P_{t+1}] + (1-\tau)R_t - (1+r+\delta)P_t}{\lambda\sigma^2}$, and the house price is given by $P_t = \frac{E_t[P_{t+1}] + \hat{R} - \tau R_t}{1+r+\delta}$. The two fitness functions modify to $A_t^E = (P_{t-1} + (1-\tau)R_{t-2} - (1+r+\delta)P_{t-2})Z_{t-2}^E$ and $A_t^R = (P_{t-1} + (1-\tau)R_{t-2} - (1+r+\delta)P_{t-2})Z_{t-2}^R - c$. Since the other equations do not change, we arrive at the following results.

Proposition 3. *At the model's unique steady state, we have* $\overline{P} = \frac{(1-\tau)\alpha\delta}{(r+\delta)\delta + ((1-\tau)\beta + \lambda\sigma^2)\gamma} = F$, $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *implying that* $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[\nu c]}$. *Suppose that the steady state is locally asymptotically stable. If* $\overline{N}^E\chi\delta + \frac{\gamma((1-\tau)\beta + \lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ *is violated, a Neimark-Sacker bifurcation occurs.*

Proposition 3 reveals that a tax on rental income has a similar effect on the model's steady state as a property tax. With an increasing tax rate on rental income, investors have fewer incentives to invest in the housing market. As a consequence, the demand

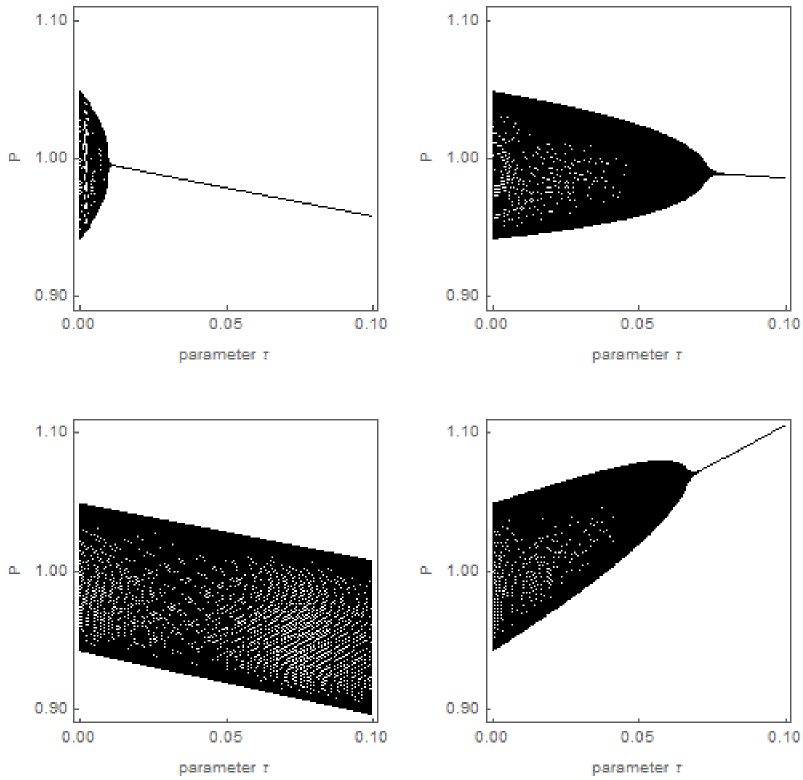

Figure 5: Effects of taxes on the value of houses, rental income, owning housing stock and revenue of housing constructors, respectively. Base parameter setting, except that τ is varied as indicated on the axis.

for housing stock is lower and the fundamental house price $\overline{P}$ declines. Therefore, the fundamental housing stock $\overline{H}$ decreases and the fundamental rent level $\overline{R}$ increases. A higher tax on rental income makes a Neimark-Sacker bifurcation also less likely. The bifurcation diagram depicted in the top right panel of Figure 5 shows that increasing values for τ make the amplitude of house price fluctuations smaller up to the point where the threshold value $\tau_{crit}^{NS} \approx 0.073$ is reached. Then, quasi-periodic dynamics turns into fixed-point dynamics.

5.3. Tax on owning housing stock

Policy makers may also consider imposing a tax on owning housing stock. Investor i's wealth equation can then be written as

$$W_{t+1}^i = (1+r)W_t^i + Z_t^i(P_{t+1} + R_t - \tau - (1+r+\delta)P_t) - c^i, \tag{25}$$

where τ is the tax rate. Investors' total demand becomes $Z_t = \frac{E_t[P_{t+1}]+R_t-\tau-(1+r+\delta)P_t}{\lambda\sigma^2}$, house prices are given by $P_t = \frac{E_t[P_{t+1}]+\hat{R}-\tau}{1+r+\delta}$, and the fitness functions turn into $A_t^E = (P_{t-1}+R_{t-2}-\tau-(1+r+\delta)P_{t-2})Z_{t-2}^E$ and $A_t^R = (P_{t-1}+R_{t-2}-\tau-(1+r+\delta)P_{t-2})Z_{t-2}^R-c$. The following proposition summarizes the main effects of such a tax.

Proposition 4. *At the model's unique steady state, we have* $\overline{P} = \frac{(\alpha-\tau)\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)\gamma} = F$, $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *implying that* $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[\nu c]}$. *Suppose that the steady state is locally asymptotically stable. If* $\overline{N}^E\chi\delta + \frac{\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ *is violated, a Neimark-Sacker bifurcation occurs.*

Proposition 4 shows that higher tax rates on owning housing stock also reduces the fundamental house price $\overline{P}$. As a result, the fundamental housing stock $\overline{H}$ decreases and the fundamental rent level $\overline{R}$ increases. However, the Neimark-Sacker stability condition reveals that it is independent of the tax rate τ. Moreover, the bifurcation diagram depicted in the bottom left panel of Figure 5 indicates that house price oscillations remain basically constant if the tax rate increases. Hence, a tax on owning housing stock merely shifts the dynamics downwards.

5.4. Revenue tax for housing constructors

Alternatively, policy makers may decide to tax housing constructors. For instance, a revenue tax for housing constructors turns their profit maximization problem into

$$max_{I_t}\{(1-\tau)E_{t-1}[P_t]I_t - C_t\}, \tag{26}$$

where τ denotes the tax rate. The optimal supply of new housing is then given by $I_t = (1-\tau)\gamma P_{t-1}$, and the housing stock evolves as $H_t = (1-\delta)H_{t-1} + (1-\tau)\gamma P_{t-1}$. Since all other equations remain unaffected by such a tax, we arrive at the following results.

Proposition 5. *At the model's unique steady state, we have* $\overline{P} = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)(1-\tau)\gamma} = F$, $\overline{H} = \frac{(1-\tau)\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *implying that* $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R =$

$\frac{1}{1+exp[\nu c]}$. *Suppose that the steady state is locally asymptotically stable. If* $\overline{N}^E\chi\delta + \frac{(1-\tau)\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ *is violated, a Neimark-Sacker bifurcation occurs.*

As stated in Proposition 5, the effects of an increasing revenue tax for housing constructors are qualitatively similar to those of a decrease in the inverse cost parameter γ, i.e. to higher building costs. This can be explained as follows. Higher tax rates make housing construction less profitable for constructors. Since fewer houses are built, the fundamental housing stock $\overline{H}$ declines. Therefore, both the fundamental house price $\overline{P}$ and the fundamental rent level $\overline{R}$ increase. A higher tax rate for housing constructors is beneficial for market stability in the sense that it may counter a Neimark-Sacker bifurcation. In fact, it becomes clear from the bifurcation diagram presented in the bottom right panel of Figure 5 that the amplitude of house price fluctuations becomes smaller as the tax rate increases. Moreover, the steady state becomes stable if τ exceeds the critical value $\tau^{NS}_{crit} \approx 0.067$.

5.5. Tax on wealth of investors

How does a wealth tax affect the dynamics of housing markets? Suppose, for simplicity, that investors' wealth is always positive. Then investor i's wealth equation is given by

$$W^i_{t+1} = (1-\tau)((1+r)W^i_t + Z^i_t(P_{t+1} + R_t - (1+r+\delta)P_t)) - c^i, \tag{27}$$

where τ represents the wealth tax rate. Note that investors' total demand for housing stock is now determined by $Z_t = \frac{E_t[P_{t+1}]+R_t-(1+r+\delta)P_t}{(1-\tau)\lambda\sigma^2}$, and house prices follow $P_t = \frac{E_t[P_{t+1}]+\hat{R}+\tau\lambda\sigma^2 H_t}{1+r+\delta}$. Furthermore, the expectation rules' fitness functions take the form $A^E_t = (1-\tau)(P_{t-1} + R_{t-2} - (1+r+\delta)P_{t-2})Z^E_{t-2}$ and $A^R_t = (1-\tau)(P_{t-1} + R_{t-2} - (1+r+\delta)P_{t-2})Z^R_{t-2} - c$. In Proposition 6, we capture the main effects of a wealth tax.

Proposition 6. *At the model's unique steady state, we have* $\overline{P} = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+(1-\tau)\lambda\sigma^2)\gamma} = F$, $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *implying that* $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[\nu c]}$. *Suppose that the steady state is locally asymptotically stable. If* $\overline{N}^E\chi\delta + \frac{\gamma(\beta+(1-\tau)\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ *is violated, a Neimark-Sacker bifurcation occurs.*

Since higher wealth taxes reduce the housing market risk for investors, their demand for housing stock increases. Consequently, the fundamental house price $\overline{P}$ and the fundamental housing stock $\overline{H}$ increase, while the fundamental rent level $\overline{R}$ decreases.

Proposition 6 also indicates that higher wealth taxes may stabilize the housing market. As illustrated by the top left panel of Figure 6, a wealth tax of $\tau_{crit}^{NS} \approx 0.734$ is needed to suppress endogenous house price fluctuations. Taking these values literally, the stability effect of a wealth tax seems to be rather weak, and the fundamental house price $\overline{P}$ rises strongly with τ. However, the strength of τ depends on the risk parameters λ and σ^2.

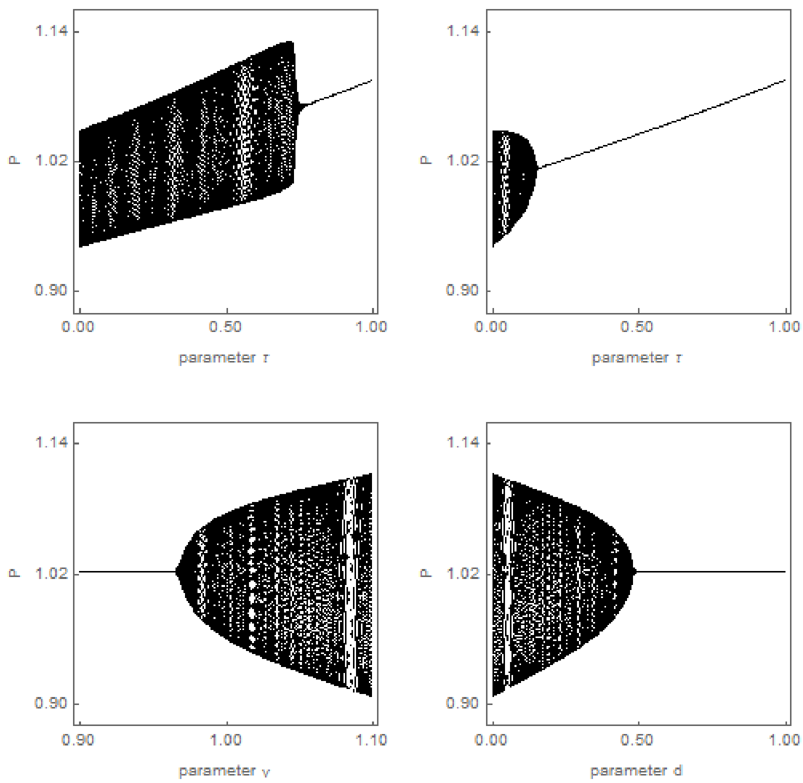

Figure 6: Effects of wealth taxes and partial deductibility of information costs. Top left: Base parameter setting, except that τ is varied as indicated on the axis. Top right: The same, except that $d = 0.25$. Bottom left: Base parameter setting, except that $\tau = 0.25$ and ν as indicated on the axis. Bottom right: Base parameter setting, except that $\tau = 0.25$, $\nu = 1.1$ and d as indicated on the axis.

5.6. Tax deductibility of information costs

Finally, we discuss the case in which the information costs associated with using the regressive expectation rule are partially deductible from wealth taxes. The wealth equation of investors opting for regressive expectations then reads

$$W_{t+1}^R = (1-\tau)((1+r)W_t + Z_t(P_{t+1} + R_t - (1+r+\delta)P_t)) - c(1-\tau d), \qquad (28)$$

where $0 \leq d \leq 1$ denotes the degree of deductibility of information costs. Compared to the scenario discussed in the previous section, the fitness function of the regressive expectation rule now becomes $A_t^R = (1-\tau)(P_{t-1}+R_{t-2}-(1+r+\delta)P_{t-2})Z_{t-2}^R-(1-\tau d)c$. This leads to the following results.

Proposition 7. *At the model's unique steady state, we have* $\overline{P} = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+(1-\tau)\lambda\sigma^2)\gamma} = F$, $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ *and* $\overline{R} = \alpha - \beta\overline{H}$, *implying that* $\overline{N}^E = \frac{1}{1+exp[-(1-\tau d)\nu c]}$ *and* $\overline{N}^R = \frac{1}{1+exp[(1-\tau d)\nu c]}$. *Suppose that the steady state is locally asymptotically stable. If* $\overline{N}^E\chi\delta + \frac{\gamma(\beta+(1-\tau)\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ *is violated, a Neimark-Sacker bifurcation occurs.*

As reported in Proposition 7, a partial tax deductibility of information costs promotes the use of the stabilizing regressive expectation rule. Note that this does not change the steady-state values $\overline{P}$, $\overline{H}$ and $\overline{R}$, but does affect the steady state's stability domain. Since the market share of extrapolative expectations $\overline{N}^E$ decreases with d, a violation of the Neimark-Sacker condition becomes less likely if a larger fraction of information costs can be deducted from tax payments. In the top right panel of Figure 6, we repeat the simulation from the top left panel, except that we set $d = 0.25$. As can be seen, increasing wealth taxes again decrease the range of house price oscillations, but the dynamics already converges towards the steady state if the tax rate exceeds $\tau_{crit}^{NS} \approx 0.151$.

The bottom left panel of Figure 6 shows a bifurcation diagram for the intensity of choice with $\tau = 0.25$ and $d = 0$. As already discussed in Section 4, increasing values of ν destabilize the system. In this case, the steady state becomes unstable, and quasi-periodic dynamics emerges as soon as $\nu \geq \nu_{crit}^{NS} \approx 0.969$. Now suppose we have a situation in which $\tau = 0.25$ and $\nu = 1.1$, and parameter d is increased, as it is illustrated in the bottom right panel. Obviously, endogenous dynamics dies out at $d = d_{crit}^{NS} \approx 0.478$, and the system settles down at its steady state. A remark is in order. The original contribution by Brock and Hommes (1997) studies a cobweb model in which firms switch between a free destabilizing and a costly stabilizing expectation rule. As the firms' intensity of choice increases, more and more of them opt for the free destabilizing expectation rule, and fixed-point dynamics turns into increasingly complex and volatile dynamics, a phenomenon that has been called a "rational route to randomness". Schmitt and Westerhoff (2015) show that a profit tax, reducing the fitness differentials between expectation rules, may reverse this outcome. Interestingly, we observe a similar phenomenon within our model. Note that the steady-state fraction of the regressive expectation rule increases with d and τ, provided that $d, \tau > 0$. For $d = 1$, any destabilizing increase in ν can be countered by an increase in τ. If possible, policy makers should promote the use of regressive expectations. Such a change in investors' behavior improves market stability, without affecting the housing market's steady-state level.

6. Conclusions

Housing markets regularly display dramatic bubbles. According to Case and Shiller (2003) and Case et al. (2013), such dynamics, which may be quite harmful for the real economy, are due to investors' optimistic house price expectations. However, Glaeser et al. (2008) argue that the real side of housing markets is also relevant for the formation and duration of bubbles. By combining Poterba's (1984, 1991) user cost model and Brock and Hommes' (1997, 1998) heuristic switching approach, we develop a novel housing market model that seeks to take these observations into account.

The real part of our model comprises a rental market and a housing capital market, and determines key relations between the house price, the housing stock and the rent level. The behavioral part of our model consists of housing market investors who switch between competing expectation rules with respect to their past performance, thereby reflecting a boundedly rational learning behavior. Amongst others, our model reveals that endogenous boom-bust housing market dynamics may arise if investors rely heavily on extrapolative expectations. Fortunately, policy makers have the opportunity to stabilize such dynamics by adjusting the tax code. For instance, a property tax or a tax on rental income tames the housing market. However, such a tax also affects the housing market's steady-state level, an aspect which should not be overlooked.

Without question, the dynamics of housing markets is driven by a complex interplay between real and behavioral forces, and a complete understanding of the functioning of housing markets is still lacking. However, we hope that our model makes some progress in this direction. Moreover, we would like to stress that our model may serve as a framework to explore how policy makers may affect the steady state, its stability and the out-of-equilibrium behavior of housing markets via adjusting the tax code. Of course, much more work is needed in this exciting and relevant research field.

Appendix

Here, we provide a detailed proof of Proposition 1. Note that the derivation of Propositions 2-7 follows along quite similar lines of reasoning. First of all, we need to express our model in form of a dynamical system. We therefore introduce the auxiliary variables $x_t = P_{t-1}$, $y_t = x_{t-1}$, $z_t = y_{t-1}$ and $k_t = H_{t-1}$. Moreover, it is helpful to use the difference in fractions, given by $m_t = N_t^R - N_t^E = Tanh\left[\frac{\nu}{2}(A_t^R - A_t^E)\right]$. Since $N_t^R + N_t^E = 1$, it follows that $N_t^E = \frac{1-m_t}{2}$ and $N_t^R = \frac{1+m_t}{2}$. The dynamical system of our model can thus be summarized by the following six-dimensional nonlinear map

$$T: \begin{cases} P_t = \frac{E_t[P_{t+1}]+\alpha-\beta(\gamma P_{t-1}+(1-\delta)H_{t-1})-\lambda\sigma^2(\gamma P_{t-1}+(1-\delta)H_{t-1})}{1+r+\delta} \\ H_t = \gamma P_{t-1} + (1-\delta)H_{t-1} \\ x_t = P_{t-1} \\ k_t = H_{t-1} \\ y_t = x_{t-1} \\ z_t = y_{t-1} \end{cases},$$

where

$$E_t[P_{t+1}] = \frac{1-m_t}{2}\big(P_{t-1} + \chi(P_{t-1} - x_{t-1})\big) + \frac{1+m_t}{2}\big(P_{t-1} + \phi(F - P_{t-1})\big)$$

and

$$m_t = Tanh\left[\frac{\nu}{2}\left\{\big(P_{t-1}+\alpha-\beta k_{t-1}-(1+r+\delta)x_{t-1}\big)\frac{\phi(F-y_{t-1})-\chi(y_{t-1}-z_{t-1})}{\lambda\sigma^2} - c\right\}\right].$$

By imposing the fact that price expectations are realized at the steady state, i.e. $E_t[\overline{P}] = \overline{P}$, implying that $\overline{P} = F$, the model's dynamical system gives rise to the unique steady state $FSS = (\overline{P}, \overline{H}, \overline{x}, \overline{k}, \overline{y}, \overline{z}) = (\overline{P}, \overline{H}, \overline{P}, \overline{H}, \overline{P}, \overline{P})$, where $\overline{P} = F = \frac{\alpha\delta}{\beta\gamma+\delta(r+\delta)+\gamma\lambda\sigma^2}$ and $\overline{H} = \overline{P}\frac{\gamma}{\delta}$. Since prices mirror their fundamental value at the steady state, we call it the fundamental steady state. Furthermore, by using $\overline{R} = \alpha - \beta\overline{H}$ we can also express steady-state prices as $\overline{P} = F = \frac{\overline{R}-\lambda\sigma^2\overline{H}}{r+\delta}$.

To explore the steady state's stability properties, we use the Jacobian matrix, computed at the fundamental steady state, i.e.

$$J(FSS) = \begin{pmatrix} \frac{2-2\gamma(\beta+\gamma\sigma^2)-\phi+\chi-(\phi+\chi)\overline{m}}{2(1+r+\delta)} & \frac{(\delta-1)(\beta+\lambda\sigma^2)}{1+r+\delta} & -\frac{\chi(1-\overline{m})}{2(1+r+\delta)} & 0 & 0 & 0 \\ \gamma & 1-\delta & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

where $\overline{m} = Tanh\left[-\frac{\nu}{2}c\right]$, and derive the characteristic polynomial

$$\kappa^3(\kappa^3 - a_1\kappa^2 - a_2\kappa - a_3) = 0,$$

where $a_1 = \frac{-4+2r(\delta-1)+2\delta^2+2\gamma(\beta+\lambda\sigma^2)+\phi+\overline{m}\phi+(\overline{m}-1)\chi}{2(1+r+\delta)}$, $a_2 = \frac{(\delta-1)(-2+\phi+\overline{m}\phi)+(\overline{m}-1)(\delta-2)\chi}{2(1+r+\delta)}$ and $a_3 = -\frac{(\overline{m}-1)(\delta-1)\chi}{2(1+r+\delta)}$. The fixed point of our model is locally asymptotically stable if and only if all six eigenvalues of the Jacobian matrix are less than one in absolute value. Note that three eigenvalues, say $\kappa_{1,2,3}$, are equal to zero, while the other three eigenvalues, say $\kappa_{4,5,6}$, result from the remaining third-degree characteristic polynomial. For this reason, we follow Lines et al. (2019), who provide a simplified set of conditions to explore a steady state's stability properties for such a problem. In fact, they show that a fixed point of a third-degree characteristic polynomial loses its stability if (I) $1 + a_1 + a_2 + a_3 > 0$, (II) $1 - a_1 + a_2 - a_3 > 0$ or (III) $1 - a_2 + a_1a_3 - a_3^2 > 0$ is violated by a continuous change of a model parameter. Moreover, a violation of (I), (II) or (III), while the other two conditions hold, is associated with a Fold, Flip and Neimark-Sacker bifurcation, respectively. In our case, tedious computations reveal that this results in

$$\text{(I)} \quad \phi(1+\overline{m})\delta > -2\big(\beta\gamma + \delta(r+\delta) + \gamma\lambda\sigma^2\big),$$

$$\text{(II)} \quad \phi(1+\overline{m}) < 2\Big(r + \frac{\gamma(\beta+\lambda\sigma^2)}{(\delta-2)} + 2 + \delta + \chi - \overline{m}\chi\Big)$$

and

$$\text{(III)} \quad (1-\overline{m})\chi\delta + \frac{2\gamma(\beta+\lambda\sigma^2)(1-\overline{m})\chi}{2(1+r+\delta)-(1-\overline{m})\chi} < (1+\overline{m})\phi + \frac{2(2\delta+r)}{1-\delta}.$$

Recall that $0 \leq \phi \leq 1$, $0 < \delta < 1$ and $\beta, \gamma, r, \lambda\sigma^2 > 0$. Also, we have $0 \leq \overline{m} \leq 1$ which implies that condition (I) is always satisfied. Finally, we use $\overline{m} = \overline{N}^R - \overline{N}^E$, $\overline{N}^E = \frac{1-\overline{m}}{2}$

and $\overline{N}^R = \frac{1+\overline{m}}{2}$, and rewrite inequalities (II) and (III) as

$$\text{(II')} \quad \overline{N}^R \phi + \frac{\gamma(\beta + \lambda\sigma^2)}{2-\delta} < 2 + r + \delta + 2\chi\overline{N}^E$$

and

$$\text{(III')} \quad \overline{N}^E \chi\delta + \frac{\gamma(\beta + \lambda\sigma^2)\overline{N}^E \chi}{1 + r + \delta - \overline{N}^E \chi} < \overline{N}^R \phi + \frac{2\delta + r}{1-\delta},$$

which correspond to (ii) and (i) in Proposition 1, respectively. For the derivation of Propositions 2-7, it is helpful to note that the tax parameter τ is always closely related to a real or behavioral model parameter, as pointed out in Section 5. For instance, in the case of a property tax, τ always appears in connection with r.

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