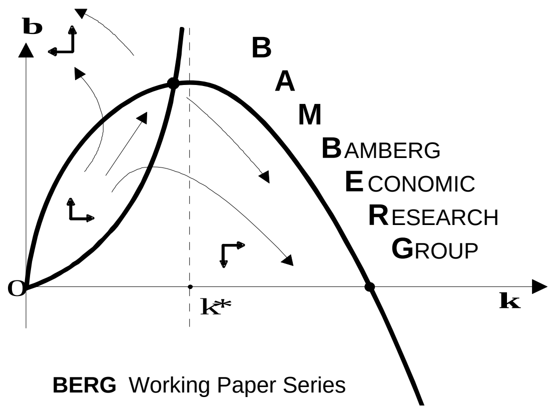
This research was carried out in the Bamberg Doctoral Research Group on Behavioral Macroeconomics (BaGBeM) supported by the Hans-Böckler Foundation (PK 045)

Traders, forecasters and financial instability: A model of individual learning of anchor-and-adjustment heuristics

Tomasz Makarewicz

Working Paper No. 141

January 2019



Bamberg Economic Research Group
Bamberg University
Feldkirchenstraße 21
D-96052 Bamberg
Telefax: (0951) 863 5547
Telephone: (0951) 863 2687
felix.stuebben@uni-bamberg.de
http://www.uni-bamberg.de/vwl/forschung/berg/

ISBN 978-3-943153-62-0

Redaktion:
Dr. Felix Stübben*

* felix.stuebben@uni-bamberg.de

Traders, forecasters and financial instability: A model of individual learning of anchor-and-adjustment heuristics.

Tomasz Makarewicz[1]

[1]Otto-Friedrich-Universität Bamberg

December 26, 2018

Abstract

Behavioral and experimental literature on financial instability focuses on either subjective price expectations (Learning-to-Forecast experiments) or individual trading (Learning-to-Optimize experiments). Bao et al. (2017) have shown that subjects have problems with both tasks. In this paper, I explore these experimental results by investigating a model in which financial traders individually learn how to use forecasting and/or trading anchor-and-adjustment heuristics by updating them with Genetic Algorithms.

The model replicates the main outcomes of these two threads of the experimental finance literature. It shows that both forecasters and traders coordinate on chasing asset price trends, which in turn causes substantial and self-fulfilling price oscillations, albeit larger and faster in the case of trading markets. When agents have to learn both tasks, financial instability becomes more persistent.

JEL codes: C53, C63, C91, D03, D83, D84.

Keywords: Financial Instability, Learning-to-Forecast and Learning-to-Optimize Experiments, Genetic Algorithm Model of Individual Learning.

1 Introduction

The financial meltdown of 2007 and the resulting Great Recession demonstrated the importance of studying the causes and effects of financial instability. One famous example is the S&P500 stock index, which fell from 1565.15 points on 9th October 2007 to 676.53 points on 9th March 2009. Afterwards, the index surged with virtually undisturbed momentum, until in January 2018 it reached its historically highest level of 2872.87 points. This means that, within a single decade after the peak of the 2007, S&P500 first lost more than half of its value, only to roughly quadruple during the recovery, beating the maximum of the first decade of the 2000s by a factor of 183.5%. NASDAQ and Frankfurt DAX indices followed a similar path of bust and boom, while many other stock indices (including Shanghai, London, Paris and Warsaw) experienced similar drops between 2007 and 2009, but recovered in a less spectacular fashion, to around pre-crisis levels. What makes such enormous price swings possible?

Modern models of asset markets make two assumptions about the behavior of financial investors. The first assumption relates the available market information with agents' forecasts of an asset price. The second assumption relates these forecasts to agents' trading positions (or demand schedules). Traditionally, the financial literature has followed Muth (1961) and embraced the Rational Expectations (RE) hypothesis: forecasting and trading are *mutually* model-consistent with each other and with the aggregate asset price mechanism. This is often used, especially in modern macroeconomics, to justify the Efficient Market Hypothesis: asset prices are an accurate reflection of their fundamentals (including the individual investors' risk attitude). However, it is difficult to imagine that, within a single decade, the fundamentals of one stock index could drop by half and then quadruple. A simpler explanation of the crisis is to assume that financial investors, regardless of how smart they are, face constraints on their cognitive abilities and can commit forecasting and investment mistakes (Barberis and Thaler, 2003; Shiller, 1981). As a result, markets do not necessarily have to stabilize at their fundamental level. This leads to the central question of this paper: *is financial instability caused by non-rational expectations, or by agents' inability to transform expectations into rational trading positions – or maybe by both?*

This question cannot be easily answered by an empirical study, since many relevant financial variables, in particular fundamental values or investors' forecasts, are not directly observable. One popular approach to circumventing this issue is to run a laboratory experiment, where the researcher can directly control the structure and parametrization of the market, and ask subjects to elicit their beliefs (Hommes, 2011). The literature on financial experiments can be roughly divided into two main threads: "Learning-to-Forecast" (LtF), which focuses on the subjects' expectation formation (Bao et al., 2012; Colasante et al., 2018; Heemeijer et al., 2009; Hommes et al., 2007; Marimon et al., 1993); and "Learning-to-Optimize", where subjects are directly tasked with trading an asset (for a selection of examples from this vast body of literature, see Breaban and Noussair, 2015; Dufwenberg et al., 2005; Kirchler et al., 2012; Lei et al., 2001; Noussair and Tucker, 2013; Smith et al., 1988; Weber et al., 2018). These two branches of the experimental literature yield similar stylized facts: subjects more often than not coordinate on speculative bubbles and market crashes (Palan, 2013).[1] In experiments with a constant fundamental price, this behavior can lead to repeated off-fundamental price oscillations (see Hommes et al., 2005, for an example).

A second experimental pattern is that subjects do not necessarily have to converge to the rational expectations solution. Nevertheless, they do rely on smart and successful behavioral rules, which can often be represented as simple *anchor-and-adjustment* heuristics (Northcraft and Neale, 1987; Sewell, 2007; Tversky and Kahneman, 1974). Moreover, subjects are smart in how they choose their heuristics. For the example of LtF experiments, when subjects observe a price trend, they will try to chase it. In this case, they use the last observed price as an anchor, which then they adjust by the last observed price difference (Hommes, 2011; Palan, 2013). They furthermore adjust the degree of trend chasing depending on the particular market dynamics that they experience (Anufriev et al., 2018). Similarly, when asked to trade, subjects often adjust their previous position in hight of the expected or last observed return of the asset (Bao et al., 2017).

There is only a scarce number of experimental studies that have tried to combine forecasting and trading designs. One notable example is a paper by Nickerson et al. (2007), who enhanced the LtO design of Smith et al. (1988) by eliciting subjects' price forecasts. The authors, in line with LtF experiments, found that subjects extrapolated observed price trends. These results have been further investigated by Bao et al. (2017), who focused directly on the difference between forecasting and trading experimental designs. The authors used a simple asset market and asked their subjects to elicit forecasts (LtF treatment, where the market price depended on the average price forecast), directly trade an asset (LtO treatment, where the price depended on the average quantity decision), or perform both tasks (Mixed treatment, where subjects were rewarded for both forecasting and trading efficiency, but the price solely depended on quantity decisions). Conforming the previous literature, the authors found that all three treatments led to substantial market instability, but the two trading treatments were relatively more unstable. Furthermore, the forecasting treatment yielded largely homogeneous dynamics, whereas the groups in the two trading treatments coordinate on price oscillations with diversified amplitude and period.

In other words, the results of the experiment by Bao et al. (2017) suggest that people have problems with both forecasting and optimal trading, but the latter has a more destabilizing effect on the market. In response, subjects learn to coordinate on anchor-and-adjustment rules with a more aggressive adjustment factor under trading treatments. In fact, Bao et al. (2017) showed that only one quarter of their subjects traded consistently with their forecasts under the Mixed treatment (where both decisions were observed). This result echoes the literature on experimental game theory, where some papers have demonstrated that subjects do not have to act consistently with their beliefs (for a number of examples, see Costa-Gomes and

[1] This also holds for experiments in which rational bubbles are impossible due to the mathematical structure of the experimental economy; see Heemeijer et al. (2009) for an example.

Weizsäcker, 2008; Crawford et al., 2013; Duffy and Tavits, 2008; Rutström and Wilcox, 2009).

The most prominent strand of the theoretical literature that tries to explain the experimental evidence is based on the Heuristic Switching Model (henceforth HSM for short; see Anufriev and Hommes, 2012; Brock and Hommes, 1998). The idea being this model is that agents rely on simple *heuristics*, rules of thumb (such as fundamental forecasts, naive and adaptive expectations, or trend-following rules), and they tend to switch to rules with higher past forecasting accuracy. Under some circumstances, investors may converge to the fundamental price, but often they coordinate on trend-chasing behavior, which leads to persistent oscillatory, explosive or chaotic dynamics (see Hommes, 2013a, for a literature overview and examples). The reason is the positive feedback between expectations and trades on the one hand, and realized prices on the other (Hommes, 2013b). For example, if investors hold optimistic beliefs about the future prices of an asset, they will buy it, which drives its price up. As a result, investor sentiment is self-fulfilling, and agents can easily coordinate on price trends. A plethora of experimental and theoretical investigations have shown that this behavioral approach can explain many financial stylized facts, and outperforms the RE model in empirical studies, in particular for housing markets (Bolt et al., 2014; Dieci et al., 2018; Kouwenberg et al., 2010) as well markets as various other financial assets (Boswijk et al., 2007; Dieci and Westerhoff, 2010, 2012; Frijns et al., 2010; Hommes and Wagener, 2009; LeBaron, 2006, 2012; Lux, 1995, 2012; Westerhoff and Reitz, 2003). Finally, HSM can be grounded in an explicit model of individual learning (Anufriev et al., 2018). In particular, the famous model of the Santa Fe Artificial Stock Market demonstrates that technical trading (akin to trend-chasing behavior) is a likely outcome of generalized learning dynamics (Arthur, 2018).

To the best of my knowledge, there is no "trading" counterpart to the HSM literature. In particular, there exists no widely acknowledged model where heterogeneous agents learn how to trade financial assets with some anchor-and-adjustment heuristics. In practice, the literature on behavioral finance assumes one of the two extremes: agents trade randomly or near-randomly (see, for instance, Duffy and Ünver, 2006; Gode and Sunder, 1993); or that their only cognitive limitation lies in the forecasting problem, but then they trade consistently with their behavioral expectations, as in HSM and its derivatives (Anufriev et al., 2013a; Bao et al., 2018).

The goal of this paper is to provide a computational heterogeneous agent model that can (1) replicate the stylized facts of the LtF and LtO experiments, and in particular the experiment by Bao et al. (2017), (2) explain and interpret the results of that experiment, and (3) provide a theoretical framework for models in which both forecasting and trading decision-making are subject to bounded rationality. I will follow the work of Anufriev et al. (2018), who used a heterogeneous agent model of explicit learning to show that HSM is a good approximation of subject behavior in a number of LtF experiments. In the model presented in this paper, agents forecast and/or trade the asset with simple anchor-and-adjustment heuristics. Forecasters use the first-order heuristic as in Anufriev et al. (2018), while the trading heuristic is based on the estimated behavior of subjects from the two trading treatments of Bao et al.

(2017). Agents *independently* update their anchor-and-adjustment heuristics with the use of Genetic Algorithms (henceforth GA). This yields endogenous learning dynamics, in which the willingness of GA agents to chase the market trend forms a self-reinforcing feedback loop with unstable price dynamics.[2]

Compared to alternative models of bounded rationality and individual learning, Genetic Algorithms offer a substantial advantage: they tend to fit well to the speed and dynamics of learning that we observe in *diversified* experimental data (Arifovic and Duffy, 2018). One good reason is that, unlike many competing models, they can be easily adapted to a rich class of economic environments. The same baseline learning algorithm may be used to describe agents facing forecasting tasks, quantity decision tasks, strategic interaction problems, or a mixture of these tasks. For the example of this paper, agents use exactly the same mechanism to update forecasting and trading heuristics. This represents the degree of versatility of the actual experimental subjects and market agents well. The Genetic Algorithm model should be understood as an approximation, rather than an actual representation of our learning, but it is sufficient to provide us with a good idea of what can be expected from economic agents, and what the major driving forces of their learning are (Anufriev et al., 2018; Arthur, 2018; Dawid, 1996).

The GA model presented in this paper allows for a comparison of the behavior of forecasters, traders, and forecaster-traders of two levels of sophistication. The paper yields four important results. First, the model replicates the findings of the two threads of the behavioral finance literature on the LtF and the LtO experiments. In particular, it is able to reproduce the behavior of the experimental groups from Bao et al. (2017). It shows that both markets with forecasters and with traders generate unstable price paths. On the other hand, traders can coordinate on price oscillations with more varied amplitude and frequency than forecasters, with record bubbles even reaching 350% of the fundamental price, like in the experiment. In addition, the model replicates the surprising finding by Bao et al. (2017) that subjects may trade inconsistently with their beliefs.

The second contribution of the paper is that it demonstrates that the insights of the LtF literature are valid in the context of the LtO setup. When forecasters try to learn how to forecast the price of an asset and traders how to trade that asset, both are easily pushed towards chasing market dynamics: price trends in the case of forecasters and asset returns in the case of traders. Asset markets are positive feedback systems, where investors' optimism and bullish price dynamics reinforce each other – and the GA model demonstrates that this mechanism operates both through boundedly rational forecasting *and* trading.

The third contribution is an important lesson about a reinforcement mechanism between trading and forecasting biases. As explained above, both traders and forecasters learn to

[2]Genetic Algorithms were initially used in models where agents optimize their final *strategy* (such as forecast or quantity) instead of a *strategy rule* (such as forecasting or quantity heuristics). Important early examples can be found in Arifovic (1995) and Arifovic (1996), whereas Dawid (1996) and Vriend (2000) discussed the difference between social and individual learning applications of GAs. The rule optimization approach to GAs was used in Hommes and Lux (2013). See Anufriev et al. (2018) for a comprehensive literature overview.

chase price dynamics. But if agents are asked to perform both tasks at the same time, the two corresponding learning processes amplify each other's trend-chasing bias through the positive feedback nature of asset markets. These agents are therefore even more likely to shy away from rational expectations and trading consistently with their beliefs. Instead, they become over-reactive and can coordinate on "super-bubbles", which cause prices to oscillate between 10% and 350% of the fundamental price. In other words, *when agents have to learn both to forecast and to trade, the positive feedback of the asset market is magnified.*

The paper furthermore explores a model with both forecasters and traders, which leads to its fourth contribution. When sufficient number of traders enters the market, a tipping point occurs in which agents switch to heuristics that extrapolate market dynamics in a more aggressive fashion, thus generating larger price oscillations. In sum, the GA model presented in this paper shows that, due the positive feedback of the asset market, when agents try to learn trading and forecasting at the same time – which is what investors in empirical financial markets have to do – they are likely to be pushed far from the rational solution, possibly further than was previously presumed in the literature on behavioral finance.

This is an important warning for policy-makers. As mentioned in the first paragraph, important stock indices have recently experienced enormous swings, such as the S&P500 index, which collapsed by half and then quadrupled between 2008 and 2018. The consensus in the behavioral finance literature is that such shifts can be explained by learning dynamics, even if the underlying fundamentals are relatively stable. The contribution of this paper is to corroborate this insight, and to show that behavioral models can predict the actual order of magnitude and the persistence of the stocks' boom and bust cycle.

The paper is organized as follows. Section 2 presents the asset market and the details of experimental results of Bao et al. (2017). Section 3 discusses the model: what the forecasting and trading heuristics of agents are, and how they use Genetic Algorithms to learn. Section 4 shows the baseline results of the model for the case when either all agents are forecasters, or all agents are traders; and how these markets relate to the experimental results. Section 5 examines the link between market dynamics and the learning of GA forecasters and traders more deeply. Section 6 briefly investigates model dynamics when the market contains both forecasters and traders. The concluding section summarizes the results and the paper's contribution. Appendices provide technical details on the timing of the model, Genetic Algorithms, and model initialization.

2 The asset market

2.1 The experimental economy

Consider a set of $I = 6$ myopic investors,[3] who trade on a period to period basis. At period t,the preferences of agent i are given by the mean-variance utility function of the form

$$U_{i,t} = z_{i,t}\rho_t - \frac{a}{2}z_{i,t}^2, \tag{1}$$

where $z_{i,t}$ denotes the position of agent i at period t (note that short positions with $z_{i,t} < 0$ are possible), $a = 6$ denotes the risk aversion factor of the agent, $z_{i,t}^2$ corresponds to the perceived risk of the agent's position, and finally

$$\rho_t \equiv p_t + y - Rp_{t-1} \tag{2}$$

is the economic return of the asset, given the realized price p_t at period t, a fixed dividend $y = 3.3$ and a gross interest rate $R = 1 + r = 1.05$. For simplicity, suppose that agents face no additional liquidity constraints. The problem of agent i is that she needs to decide on $z_{i,t}$ before p_t is actually realized. Denote her forecast of p_t as $p_{i,t}^e$, then the optimal demand conditional on this belief is given by

$$z_{i,t}^* \equiv (1/6)\rho_{i,t}^e \equiv \frac{p_{i,t}^e + 3.3 - 1.05p_{t-1}}{6}. \tag{3}$$

Once all agents have submitted their individual demands, the market maker adjusts the price according to the aggregate demand with the price adjustment rule

$$p_t = p_{t-1} + \lambda \sum_i z_{i,t} + \varepsilon_t, \tag{4}$$

where $\lambda = 1/R = 20/21$ is a scaling factor, and $\varepsilon_t \sim NID(0, 1)$ is a small price shock. If agents trade consistently with their price expectations, then under experimental parametrization, price equation (4) reduces to

$$p_t = p^f + \frac{20}{21}\left(\bar{p}_t^e - p^f\right) + \varepsilon_t, \tag{5}$$

where $p^f = y/r = 66$ is the fundamental price, and $\bar{p}_t^e \equiv (1/6)\sum_i p_{i,t}^e$ is agents' average forecast.

Straightforward derivations prove that p^f is the unique fixed point of the expected price equation (5), and hence the unique stationary RE steady state. Furthermore, because the current price p_t depends only on its own forecast through a simple linear equation, one can easily

[3] The parametrization of the economy is taken directly from the design of Bao et al. (2017) in order to have a natural empirical benchmark for the model simulations.

see that the fundamental price p^f is in fact the unique RE solution, while rational bubbles are excluded from this setup. This implies that the model only yields any real dynamics if agents fail to form perfectly rational expectations or to trade consistently with their expectations.

2.2 Experimental design and results

Bao et al. (2017) studied three treatments: Learning to Forecast (LtF), Learning to Optimize (LtO) and Mixed, each with eight groups with six subjects per group (one subject associated with one computer trader). In all three treatments, subjects were told that their task was to be a forecasting and/or trading advisor to a pension fund. They were only given a qualitative description of the price mechanism, and could only observe the aggregate market outcomes (past prices) and their individual decisions and payoffs, but not the other subjects' decisions or payoffs. They were also unaware of the number of market participants, meaning that they behaved as price-takers. Each experimental market lasted for $T = 50$ periods.

Under the LtF treatment, each subject played the role of a financial advisor to a computerized investor, who, in turn, used the subject's forecast to compute its optimal demand (3). In practice, subjects were asked to elicit price forecasts $p^e_{i,t}$, and their average forecast $\bar{p}^e_t$ was substituted directly into the law of motion (5).[4] Subjects observed the realized price p_t and were paid according to the squared error of their forecasts.

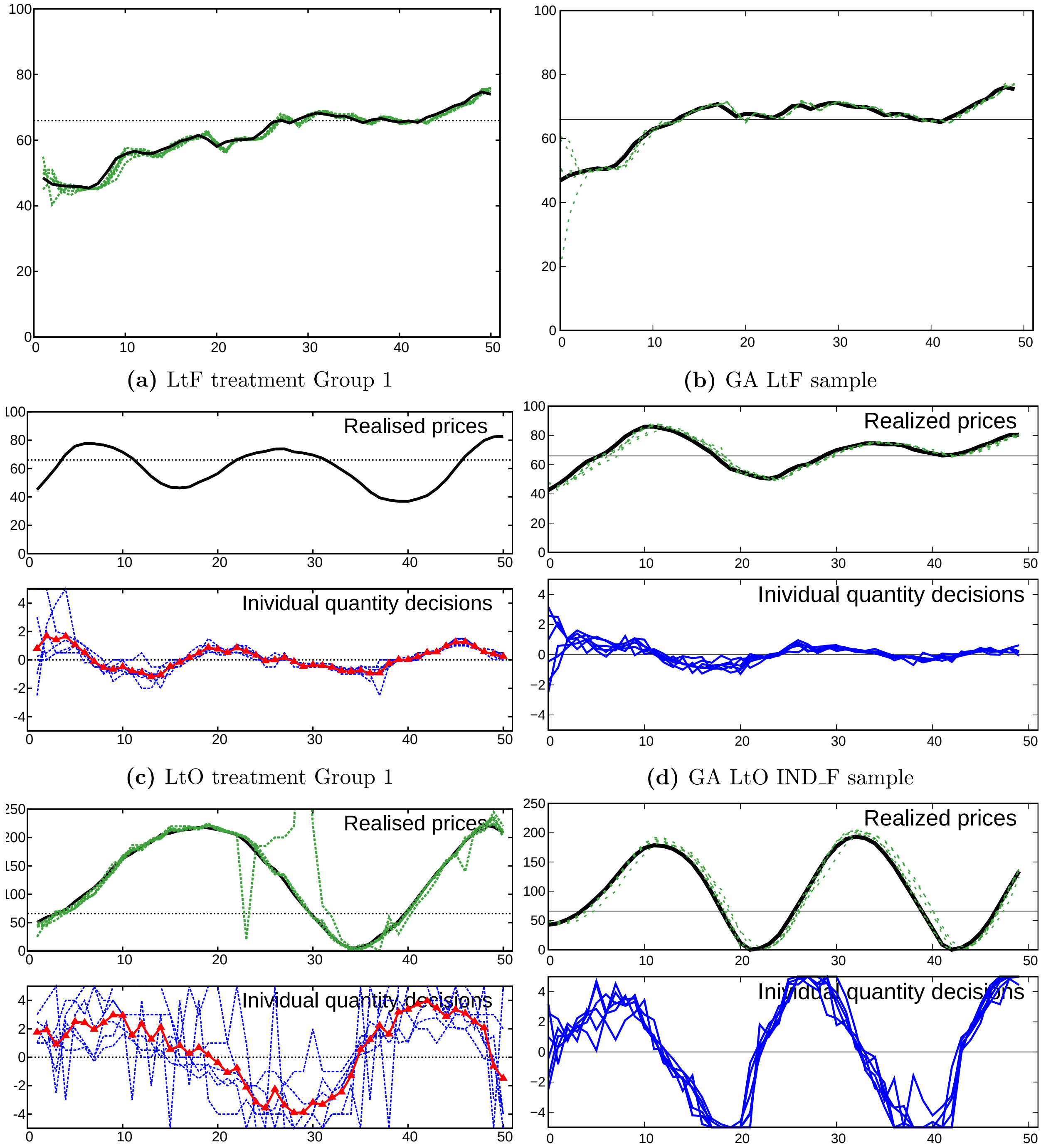
Under the LtO and Mixed treatments, subjects directly played the role of traders of the pension fund. They were provided with the mathematical formula for utility function (1). They were also given a calculator that computed the expected asset return for any price forecast, given the price in the previous period, as well as a table that related the expected asset return and trading position to the investors' utility. Subjects were tasked with submitting their trades $z_{i,t}$, which were substituted into the price adjustment equation (4).[5] Subjects observed the realized price and utility, and moved to the next period. In the LtO treatment, they were paid based on the realized utility of their trades (1).

Under the Mixed treatment, subjects were also explicitly asked to provide their price forecasts $p^e_{i,t}$. These forecasts had no bearing on the market price. However, by the end of the experiment, the experimenters randomly chose (separately for each session) whether to pay subjects based solely on their forecasting accuracy, or solely on their trading efficiency, so that subjects had an incentive to perform both task as well as possible.

Utility function (1) is a square function of the forecasting error (see Bao et al., 2017, for discussion), and therefore under perfect rationality, the three treatments are equivalent and should lead to exactly the same dynamics; specifically perfectly rational subjects would immediately jump to the fundamental price. This was not the case in the experiment, however. Figure 1 shows one sample group per treatment. Under the LtF treatment, all eight groups

[4] In addition, price forecasts were constrained to remain in the $[p_{t-1} - 30, p_{t-1} + 30]$ interval. In practice, neither subjects nor GA agents hit these boundaries.

[5] In addition, to keep markets relatively stable, subject trades were constrained to remain in the $z_{i,t} \in [-5, 5]$ interval, which roughly corresponds to the forecast constraint under the LtF treatment. Unlike in the forecasting treatment, these trading boundaries were repeatedly hit by both subjects and GA agents.



(a) LtF treatment Group 1 **(b)** GA LtF sample

(c) LtO treatment Group 1 **(d)** GA LtO IND_F sample

(e) Mixed treatment Group 8 **(f)** GA LtO OPT_F sample

Figure 1: Left panels: sample groups from the experiment by Bao et al. (2017), one group per treatment. Right panels: sample Genetic Algorithm model simulations (see Section 3 for a definition of the model variants). The top panels show realized prices and individual forecasts under the forecasting treatment. The middle and bottom panels show results from trading treatments, where the upper part of each panel shows realized prices and individual forecasts (with the exception of c, which reports an experimental LtO group without elicited forecasts), while the bottom part of each panel shows subject/agent trades. A thick black line denotes the realized price, a thin dashed black line denotes the fundamental price $p^f = 66$, dashed green lines denote individual forecasts $p^e_{i,t}$, dotted blue lines denote individual trades $z_{i,t}$, and the red line with triangles denotes the average trade.

exhibited dynamics similar to Group 1, displayed on Figure 1a: subjects quickly coordinated on similar forecasts, but the market mildly oscillated instead of settling on the fundamental price.

The dynamics under the LtO and Mixed treatments were statistically indistinguishable, but also much more varied. Most of the groups oscillated like the two sample LtO and Mixed groups reported in Figures 1c and 1e respectively, but the specific amplitude and frequency changed from group to group. The two trading treatments were significantly more unstable than the LtF treatment. Furthermore, under the Mixed treatment, two groups coordinated on "super-bubbles" when the price reached approximately 350% of the fundamental price, only to fall to a level close to 10% of the fundamental (seen on Figure 1e).

This leads us to the three stylized facts of the experiment by Bao et al. (2017) that I want to address with my model:

S1: Trading leads to more unstable dynamics than forecasting.

S2: Trading groups generate diverse price oscillations (varying period and amplitude).

S3: Exceptionally large oscillations can emerge under trading.

3 The Genetic Algorithm model of individual learning

The agent structure of the Genetic Algorithm (GA) model is based directly on the model of Anufriev et al. (2018) and on the information and market structure of the experiment by Bao et al. (2017). Consider a set of $I = 6$ agents (referred to as GA agents) who are asked either to forecast, to trade, or to perform both task, in the asset market discussed in Section 2.1. The agents are unable to immediately form perfectly rational forecasts and trading positions, and instead they follow simple anchor-and-adjustment heuristics. They learn how to use these heuristics by updating them with a GA procedure, which constitutes *the core learning mechanism of the model.* In this section, I provide an overview of this model. For the sake of clarity of presentation, many technical details have been shifted to Appendix F. See also Haupt and Haupt (2004) or Anufriev et al. (2018) for technical discussions on the core GA procedure, and Dawid (1996) and Arifovic (1995) for sample economic applications.

3.1 Learning to forecast and learning to optimize

3.1.1 Forecasting heuristic

The forecasting heuristic of the GA agents is taken directly from Anufriev et al. (2018). Suppose that an agent i at period t wants to predict price p_t. Her forecast $p^e_{i,t}$ is based on a simple first-order rule (FOR)

$$\begin{aligned} p^e_{i,t} =& \alpha_{i,t} p_{t-1} + (1-\alpha_{i,t}) p^e_{i,t-1} + \beta_{i,t}(p_{t-1} - p_{t-2}) \\ =& p^e_{t-1} + \alpha_{i,t}(p_{t-1} - p^e_{i,t-1}) + \beta_{i,t}(p_{t-1} - p_{t-2}), \end{aligned} \tag{6}$$

where p_t is the price realized at period t and $p^e_{i,t-1}$ is agent's i forecast for the price at period $t-1$. Coefficients $\alpha_{i,t} \in [0,1]$ and $\beta_{i,t} \in [0,1.1]$ are the free parameters, and (as will be explained later) agent i will update them *over time* with GAs independently from the other agents.[6,7]

The top part of equation (6) shows the FOR heuristic as a sum of adaptive expectations (with weight α) and the trend extrapolation rule (with trend coefficient β). Alternatively, the bottom part of this equation shows the FOR as a simple anchor-and-adjustment rule: the price forecast $p^e_{i,t,h}$ is equal to the previous forecast, but adjusted by the forecasting error (with scaling factor α) and by the last observed price trend (with adjustment rate β). The FOR heuristic (6) has important special cases. It becomes a naive forecast when $\alpha = 1$ and $\beta = 0$, an adaptive forecast when $\alpha \neq 0$ and $\beta = 0$, and a pure trend following rule when $\alpha = 1$ and $\beta \neq 0$. The RE and fundamental forecasts can be expressed as the FOR rule when $\alpha = \beta = 0$ and $p^e_{i,t-1} = p^f$, which yields $p^e_{i,t} = p^f$.

3.2 Three approaches to trading

Under the LtO and Mixed treatments in the experiment by Bao et al. (2017), subjects were asked to trade the asset, so they utilized a trading anchor-and-adjustment rule. Furthermore, the experimental instructions explained that subjects should trade based on the expected return of the asset, as a function of an explicit price forecast. Additionally, under the Mixed treatment, subjects were directly asked to state their forecasts. It follows that all subjects from the Mixed treatment, and possibly some from the LtO treatment, tried to forecast the asset price just as subjects from the LtF treatment did. *But what it is not immediately clear is how subjects linked forecasting with trading – if at all.* This leads to three possible trading strategies (with the descriptions preceded by abbreviations that will be used henceforth):

NON_F No forecasting: learn to use a trading heuristic, but disregard price forecasting altogether for the trading task;

IND_F Indirect forecasting: learn to use a trading heuristic with an expected asset return component, but do not additionally learn how to directly forecast prices;

[6]For the sake of clarity, I drop the two indices (i,t) from the heuristic's free parameters in the non-technical discussion of this paper. The forecasting heuristic for the **LtF** GA agents is a function of two free parameters (price and price trend weights), as well as past market variables (realized prices and agent's forecast). To keep the notation tractable, I refer to these heuristics as functions of free parameters, or as functions of past market variables, depending on the context.

[7]Price weight α is bounded by the $[0,1]$ interval by definition. The constraint on trend weight β is based on the GA-S2 specification from Anufriev et al. (2018) and is calibrated to the subjects' exhibited trend extrapolation from the experiment by Heemeijer et al. (2009).

OPT_F Optimized forecasting: learn forecasting and trading heuristics separately, and include the optimized price forecast in the trading heuristic.

The last approach – separating the forecasting and trading tasks – was likely to be the most "rational" in the experiment. However, it was also the most cognitively demanding. On the other hand, subjects were not informed about the mathematical specification of the price equation (4), so they may have found it difficult to forecast the price, or to link it with optimal trades. In such a case, the first and second approaches may be reasonable "second-best" alternatives, particularly under the LtO treatment, where forecasting accuracy is not directly rewarded.

3.2.1 NON_F traders

NON_F agent i at period t trades based on the following simple AR rule:

$$z_{i,t} = \chi_{i,t} z_{i,t-1} + \phi_{i,t} \rho_{t-1} \in [-5, 5], \tag{7}$$

where $z_{i,t-1}$ is the agent's i trade in the previous period, ρ_{t-1} is the realized asset return in the previous period as in equation (2), and $\chi_{i,t} \in [-0.5, 1]$ and $\phi_{i,t} \in [0, 0.3]$ are the free parameters that agent i is trying to learn. This heuristic is taken directly from Bao et al. (2017), who showed that it capably described the behavior of their subjects under the LtO treatment. The bounds on the two parameters were chosen to cover their experimental distributions. Furthermore, the heuristic has a simple interpretation as an anchor-and-adjustment rule, where agent's i decision is equal to her previous trade $z_{i,t-1}$ (scaled with factor χ), adjusted using the last observed asset return (with weight ϕ).

As explained earlier, under RE every, agent forecasts $p^e_{i,t} = p^f = y/(R-1)$, and so the prices becomes $p_t = p^f + \varepsilon_t = y/(R-1) + \varepsilon_t$ at every period t. Substituting this into optimal trade (3), we obtain

$$z^{RE}_{i,t,h} = \frac{p^e_{i,t} + y - Rp_{t-1}}{6} = \frac{R}{6}\varepsilon_{t-1} \sim N\left(0, R^2/36\right). \tag{8}$$

Under RE, agent's i expected trade is thus equal to zero, while realized trades are small noise terms. Hence, the AR trading heuristic approximates the RE solution with $\chi = \phi = 0$.

3.2.2 IND_F traders

IND_F agent i at period t trades based on a richer AR rule, which can be expressed as

$$z_{i,t} = \chi_{i,t} z_{i,t-1} + \phi_{i,t} \rho_{t-1} + \zeta_{i,t} \rho^e_{i,t} \in [-5, 5], \tag{9}$$

where $\chi_{i,t} \in [-0.5, 1]$, $\phi_{i,t} \in [0, 0.3]$ and $\zeta_{i,t} \in [0, 0.3]$ are free parameters, and

$$\rho^e_{i,t} = p^e_{i,t} + y - Rp_{t-1} \tag{10}$$

is the expected asset return based on price expectation $p^e_{i,t}$, which agent i (like the GA agents in the LtF environment) builds with the FOR heuristic (6). Putting this together yields

$$\begin{aligned} z_{i,t} =& \chi_{i,t} z_{i,t-1} + \phi_{i,t}\rho_{t-1} \\ & + \zeta_{i,t}\left(\alpha_{i,t}p_{t-1} + (1-\alpha_{i,t})p^e_{t-1} + \beta(p_{t-1} - p_{t-2}) + y - Rp_{t-1}\right). \end{aligned} \tag{11}$$

This heuristic, together with its parametrization, is based on the distribution of the estimated individual rules under the Mixed treatment from the experiment by Bao et al. (2017). Remark that, even though these agents use the FOR forecasting rule, they do not conceive it as a separate heuristic, and instead treat all of the equation (11) as a singular heuristic that they optimize with GAs.

Heuristic (11) is an anchor-and-adjustment rule, where previous trade $z_{i,t-1}$ plays the role of the anchor. Unlike **NON_F** traders, however, **IND_F** traders have a more sophisticated adjustment mechanism, with both a backward-looking component (the previous asset return weighted ny ϕ) and a forward-looking component (the expected asset return weighted by ζ). Thus, these agents are more flexible, and have a more straightforward way to converge to the rational solution. Just like in the case of the LtF agents, $\alpha = \beta = 0$ and $p^e_{i,t-1} = p^f$ together yield the RE forecast. Conditional on the rational forecast, optimal demand (3) can be expressed with trading heuristic (9) when $\chi = \phi = 0$ and $\zeta = 1/6$.

3.2.3 OPT_F traders

OPT_F agent i first forecasts next price p_t using forecasting heuristic (6), like **LtF** agents. She uses them to generate an asset return forecast $\rho^e_{i,t}$ as in equation (10). Next, she trades according to a trading rule expressed as

$$\begin{aligned} z_{i,t} =& \chi_{i,t} z_{i,t-1} + \phi_{i,t}\rho_{t-1} + \zeta_{i,t}\rho^e_{i,t} \\ =& \chi_{i,t} z_{i,t-1} + \phi_{i,t}\rho_{t-1} + \zeta_{i,t}\left(p^e_{i,t} + y - Rp_{t-1}\right). \end{aligned} \tag{12}$$

Note that the trading heuristic (12) is similar to heuristic (9) of **IND_F** traders. In particular, it has the same anchor-and-adjustment interpretation, as well as the RE special case. However, **OPT_F** traders perceive their task as a two-dimensional problem, and learn *separately* how to use the forecasting rule (6) and the trading rule (12).

3.3 Genetic Algorithm

The premise of the model is that agents learn how to use different anchor-and-adjustment heuristics *via* the popular Genetic Algorithm (GA) optimization procedure. GAs are a class of numerical stochastic optimization algorithms, which are inspired by the evolutionary mechanism by which the genome of living organisms adapts to the ever-changing environment through procreation and mutation. GAs tend to outperform traditional algorithms when dealing with highly non-linear and multidimensional optimization problems, as well as with problems with non-real arguments (such as binary or integer arguments); they are popular in engineering and related fields (Haupt and Haupt, 2004).

Suppose that you want to maximize a function $V(\theta)$ of some argument set θ. In my model, V is the forecasting or trading success of a heuristic based on its parameters. The standard binary GA algorithm operates as an evolutionary search mechanism upon the grid of arguments θ in the following way. Initially, a population of H arguments (denoted as chromosomes) is created at random, where each argument is encoded with binary strings of 0s and 1s.[8] Then, at every iteration, the arguments undergo the following set of four operators:

Procreation: A set of H child (new) chromosomes is sampled from the set of parent (old) chromosomes (with replacement), where the probability associated with a parent chromosome is a monotonically increasing transformation (e.g., logistic map) of its value $V(\cdot)$ relative to value $V(\cdot)$ of other chromosomes (i.e., relative fitness).

Crossover: The child chromosomes are divided into pairs, and each pair will exchange a predetermined subset of bits with probability δ_C. Typically, this means exchanging subsets of arguments.

Mutation: With probability δ_M, every bit in every argument is swapped (1s into 0s, and vice versa).

Election: The value $V(\cdot)$ of every child is computed. If the child argument has a higher value than its parent, it replaces the parent; otherwise the parent is retained in the population, and the child disappears (see Arifovic, 1995, 1996).

The procedure runs for a predetermined number of iterations, or until a certain convergence criterion is met.

The procreation operator functions as the evolutionary mechanism, with which the better arguments slowly replace the worse ones. The two evolutionary operators – crossover and mutation – allow for experimentation of the argument value. Finally, the election operator (introduced by Arifovic, 1995) screens off experimentation that leads to poorer fit. Furthermore, these operators can be reinterpreted as a learning mechanism: agents focus on more successful strategies, experiment with their parameters, and discard unsuccessful “mutants” (see Arifovic and Duffy, 2018, for a dicussion).

This is precisely the way in which the agents in my model use GAs to learn. Before the market starts operating, every GA agent is endowed with $H = 20$ forecasting heuristics (6), or $H = 20$ trading heuristics (7) or (9), depending on her type. The exception is **OPT_F** agents, who each have $H = 20$ forecasting heuristics (6) and $K = 20$ trading heuristics (12). At the beginning of every period afterwards, when they observe the new realized price, GA

[8]GAs can operate on both floating and binary argument encoding. In this paper, the binary specification was chosen for two reasons. First, my model is directly based on Anufriev et al. (2018) and Hommes and Lux (2013). Second, binary representation allows for a more flexible set of arguments, and for a more efficient mutation operator. See also Anufriev et al. (2018) for a discussion.

agents use *one* iteration of GAs to update the free parameters of their heuristics, which are represented by binary strings with 20 bits per parameter.[9]

3.4 Sequence of events on the market

There are four baseline variants of the model, one with forecasters and three with traders, but they all have the same timing. In the first variant, denoted as **LtF**, GA agents are repeatedly asked to forecast the price, and their forecasts $p^e_{i,t}$ are used to generate the next price via the expectation-price feedback mechanism (5). These agents use the FOR heuristic (6). In the three trading variants of the model, denoted as **LtO**, GA agents are repeatedly asked to specify their trading positions $z_{i,t}$, which are used to generate the next price via the price adjustment mechanism (4). In this section, I investigate three versions of the **LtO** variant: where all GA agents are (i) homogeneously **NON_F** traders, (ii) homogeneously **IND_F** traders and (iii) homogeneously **OPT_F** traders. Regardless of the variant of the model, the market operates for $t \in \{1, \dots, 50\}$ periods as in the experiment.

At the beginning of period $t = 1$, both experimental subjects and the GA agents know no history of the market, and their initial decisions are therefore random. Following Anufriev et al. (2018), I take the initial forecasts $p^e_{i,1}$ and/or trades $z_{i,1}$ of GA agents as exogenous, sampled from a distribution calibrated to the empirical distribution of the initial forecasts and trades from Bao et al. (2017).[10]

Starting from period $t \geqslant 2$, GA agents observe the previous price p_{t-1}, and can use their heuristics. Following Anufriev et al. (2018), agents have so little information at the beginning of the session that, at period $t = 2$, that they will simply sample initial heuristics at random, so they can start with a good spread of potential strategies. In particular, they generate their heuristics from a uniform distribution, i.e. every bit in every heuristic of every agent becomes one or zero with an equal probability of 0.5.[11]

Once the agents have their heuristics initialized, the market at every period $t \geqslant 2$ operates in the following way:

1. GA agents observe the realized market price from the previous period p_{t-1}, and compute the relevant auxiliary variables such as ρ_{t-1} for the trading agents.
2. If $t \geqslant 3$, every GA agent i updates her heuristics *independently from the other agents* with *one* iteration of GA operators (procreation, mutation, crossover, election), where

[9] The full parametrization of the GAs can be found in Appendix A. I took it directly from Anufriev et al. (2018), who use Monte Carlo simulations to show it to be robust to reasonable changes of the parameters.

[10] As in the experimental design of Bao et al. (2017), the initial price in **LtO** markets is set to $p_0 = 42$ (approximately 64% of the fundamental price).

[11] Forecasting heuristic (6) also requires the initial price trend, and it is assumed, following Anufriev et al. (2018), that agents observe $\Delta p_1 = p_1 - p_0 = 0$. On the other hand, in the **LtO** variant of the model, agents are assumed to observe the first realized asset return $\rho_1 = \rho(p_1, p_0)$, so that they can use it in their trading heuristics. Auxiliary simulations show that, in fact, this detail of the model initialization plays no role, and if we set the initial observed asset return to $\rho_1 = 0$ (which roughly corresponds to $\Delta p_1 = 0$), the Monte Carlo distribution of the LtO variant remains qualitatively intact. This is because the typical initial asset return is close to zero anyway, whereas GA agents use random asset return weight ϕ.

the value function for GAs is the logit transformation of the relevant *hypothetical past performance* function $V(\cdot)$:

- **Forecasting heuristic** (6) for **LtF** and **LtO OPT_F** GA agents: hypothetical forecasting squared error in the previous period

$$V = -\left(p_{t-1} - p^e_{i,t-1}(\cdot)\right)^2, \tag{13}$$

where $p^e_{i,t-1}(\cdot)$ is a function of weights α and β.

- **Trading heuristics** for the appropriate **LtO** GA agents: hypothetical trading efficiency in the previous period

$$V = -\left(\rho_{t-1}/6 - z_{i,t-1}(\cdot)\right)^2 = \left(z^*_{t-1} - z_{i,t-1}(\cdot)\right)^2,$$

where $z_{i,t-1}(\cdot)$ is a function of weights as defined in equations (7), (9) and (12) for the three types of traders, respectively.

3. Each GA agent i samples one relevant heuristic from the list of updated rules, and uses it to generate her decision(s): forecast and/or trade. The sampling weights are exactly as in the reproduction step, namely logit transformation of the relevant hypothetical performance criterion $V(\cdot)$.

4. The market maker collects GA agents' decisions, and uses them to generate the next price p_t: based on individual price forecasts $p^e_{i,t}$ and price mechanism (5) in the **LtF** variant of the model, and based on individual trading positions $z_{i,t}$ and price adjustment mechanism (4) in the **LtO** variant of the model. Note that **LtO IND_F** and **OPT_F** GA traders' forecasts do not directly influence the market maker.

5. Once the new price p_t has been established, the algorithm goes back to its first step, and a new period $t + 1$ starts.

The market operates according to this algorithm for a pre-specified number of $T = 50$ periods, as in the experiment by Bao et al. (2017).

As explained above, GA agents do not have sufficient information to use their heuristics in period $t = 1$, so they are unable to compute the hypothetical performance of these rules for that period as well. Thus, in period $t = 2$, they skip point 2 of the model timing (no heuristic update), and in point 3 (forecast and/or trade heuristic choice), they sample one heuristic with equal weights 1/20.

It is worth emphasizing that the heuristic of **IND_F** GA agents contains a price forecast – but these agents optimize that heuristic as one object, *exclusively* on the merit of its trading efficiency, and disregarding the efficiency of the forecast. In contrast, each **LtO OPT_F** agent has *two* sets of heuristics: $H = 20$ forecasting rules (6) and $K = 20$ trading rules (12). She updates and samples them *separately*, which means that she divides her task into two semi-independent parts: how to forecast efficiently, and how to trade conditional on the already fine-tuned forecast.

3.5 Simulation setup

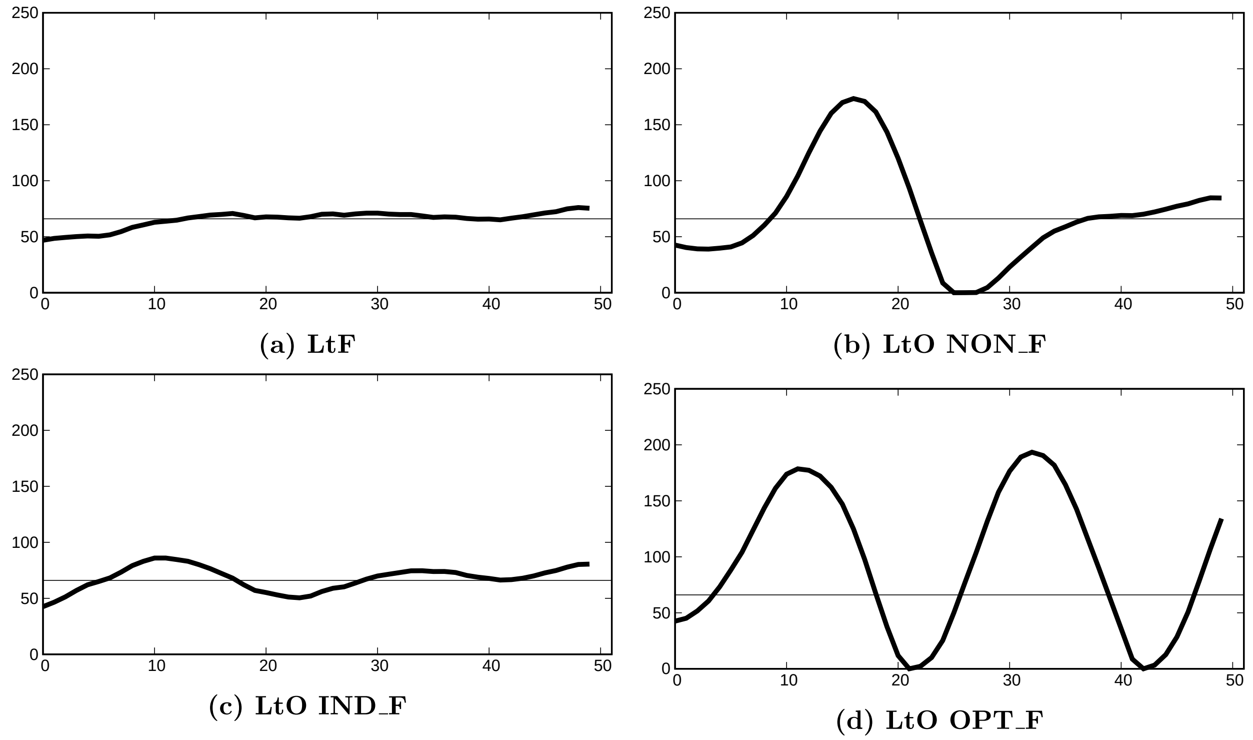
The GA model is highly non-linear and allows for no analytical solution. Instead, I focus on a simple Monte Carlo (MC) exercise with which I will study the dynamics of the model in the experimental setting of Bao et al. (2017). Two issues are worth mentioning at this point. First, the goal of this paper is to shed light on the learning that took place in the experiment, so I do not study the long-run behavior of the model. Instead, I focus on 50-period-long simulations as was the length of the experimental sessions, using the experimental data as a natural benchmark for the model results.

Second, it would be interesting to compute a measure of how well the model fits the experimental data. One example of such a measure is the one-period ahead mean squared prediction error of the model as in Anufriev et al. (2018), who showed that the GA model outperformed other forecasting models in explaining the LtF experiments. This is possible, since there exists a well-established body of behavioral literature on expectation formation, including the Heuristic Switching Model, which offers a natural benchmark for the **LtF** GA model. To the best of my knowledge, there exists no directly corresponding behavioral model of how people learn to trade financial assets, in spite of the vast body of empirical and experimental literature of what the trading biases of investors are. Therefore, computing a fitness measure for the presented model would yield little additional insight. I focus instead on the stylized facts from the experiment by Bao et al. (2017).

There are four variants of GA agents: one **LtF** and three **LtO**. In Section 4, I consider homogeneous markets with only one type of GA agents. Section 6 presents the results for heterogeneous markets with a mixture of **LtF** and **OPT_F** GA agents. For each model specification, I ran 1000 independent simulations. Each simulation was based on different initial conditions (initial forecasts and/or trades, and initial heuristics), as well as different pseudo-random numbers for the learning process.[12,13]

[12]Supply shocks ε_t in price equations (5) and (4) are constant across the simulations and taken from the experiment. GA agents use heuristics with a different number of parameters in the four model specifications. It is therefore impossible to use the same set of pseudo-random numbers for each MC study.

[13]The executable has been written in `C++` in MSVS IDE and compiled in the Windows 10 64-bit operational system. The source code and executable are available on demand.



(a) LtF **(b) LtO NON_F** **(c) LtO IND_F** **(d) LtO OPT_F**

Figure 2: Sample price paths for the GA model. The thick black line depicts the realized price, the straight thin line shows the fundamental price $p^f = 66$.

4 Results for homogeneous markets

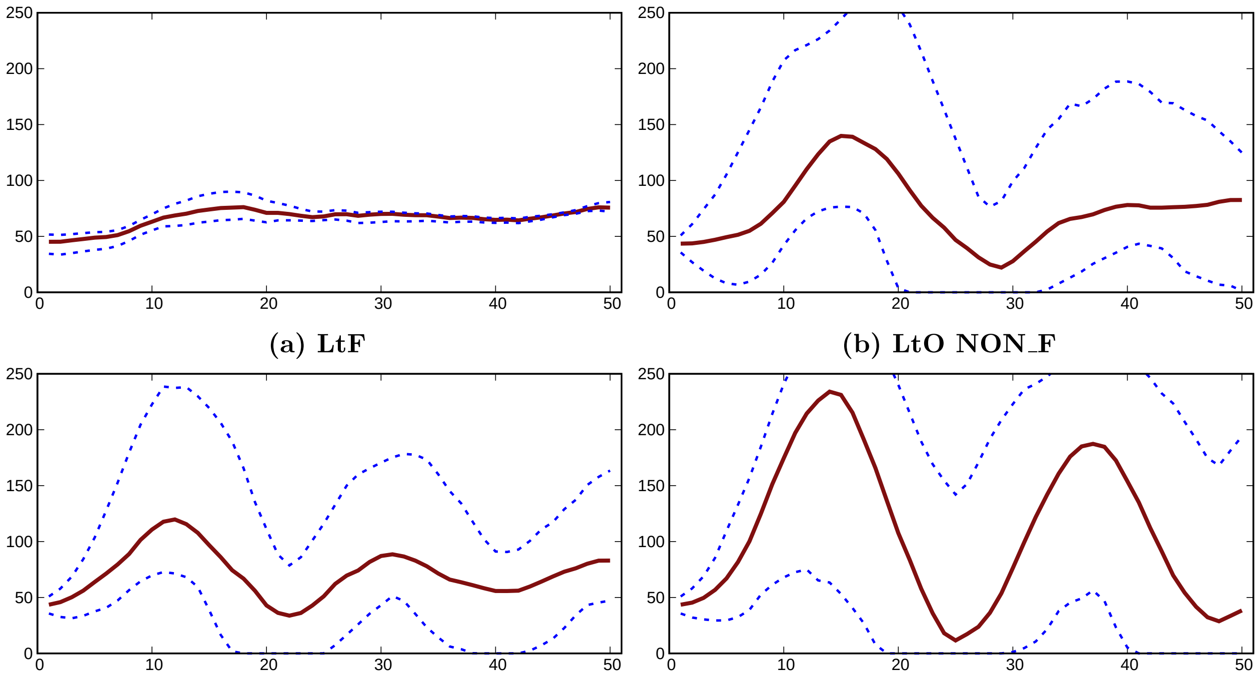
4.1 Market (in)stability

Figure 2 shows sample simulations of the four specifications of the GA model.[14] These sample paths are representative of the MC simulations of realized prices, as seen in Figure 3. In all four cases, the market does not truly converge to the RE solution; however, the three trading specifications (Figures 3b–3d) lead to substantially more pronounced price oscillations, but also to much wider 95% Confidence Intervals (CI). This result replicates the first stylized fact **S1** from the experiment by Bao et al. (2017).

All three **LtO** specifications tended to generate surprisingly regular dynamics.[15] This is visible in the similar shape of the median and the upper and lower parts of the price distribution for each model variant. These markets often brought about one or two large bubbles within 50 periods. During the first bubble, the upper bounds of the 95% CI of all three models was around $p_t = 250$, which is close to 380% of the fundamental price $p^f = 66$. The bubble was followed by a significant market crash, where the median price dropped to around half (**IND_F** GA agents) or one-third (the other two **LtO** model variants) of the fundamental, while many simulations hit the zero price bound. This in turn was followed by a renewed price increase, closing a self-sustaining cycle of booms and busts, as observed in the experiment.

[14] The sample simulations are simply the first simulation in each Monte Carlo run. Three of these, for the **LtF**, and **LtO IND_F** and **OPT_F** markets, are also presented in Figure 1.

[15] This regularity is caused by two factors: the markets use relatively similar initial forecasts and trades; and they use the same price equation disturbances ε_t.



(a) LtF **(b) LtO NON_F**

(c) LtO IND_F **(d) LtO OPT_F**

Figure 3: 50-period-ahead MC simulation (1000 markets) for the four specifications of the GA model: evolution of the asset price. The thick red line is the median and the dotted blue lines represent the 95% confidence interval for the GA model.

There were also two visible differences between the three **LtO** model specifications, as the shape of the 95% CI makes clear. First, **IND_F** markets tend to have faster oscillations. Second, **OPT_F** GA agents coordinate on bubbles having the largest amplitude, while **IND_F** agents – the smallest. Furthermore, all three **LtO** model variants generate wide 95% CIs, implying large "within-variant" heterogeneity. It follows that different **LtO** markets can generate oscillations with different dynamic properties, replicating the second stylized fact **S2** of the experiment by Bao et al. (2017).

The most unstable markets are generated by the **LtO OPT_F** model, in which GA agents include the expected asset return in their trading heuristic, and their price forecast heuristic is directly optimized. The median market generates two bubbles (Figure 3d), where the first peaks at around 234 (approximately 350% of the fundamental price $p^f = 66$), and the second at 187 (close to 300% of p^f). This *median* behavior closely resembles the two "super-bubbles" from the experiment by Bao et al. (2017), not only in terms of magnitude, but also in terms of the surprising regularity. Hence, the model replicates the third stylized fact **S3** of the experiment.

Result 1. *The 50-period-ahead simulations of the GA model replicate all three stylized facts from the experiment by Bao et al. (2017) and confirm the findings of both the Learning-to-Forecast and Learning-to-Optimize experimental literature.*

4.2 Individual learning

What is the link between the observed market dynamics and the realized individual learning in the GA model? As expected, this depends on the specification of the model, but large heterogeneity prevails even within each specification.

4.2.1 Forecasting heuristics

In line with the existing literature on the LtF setting, one important mechanism behind price oscillations is trend-chasing behavior. Figure 4 shows the Monte Carlo evolution of the two parameters from forecasting heuristic (6), price weight α and trend extrapolation coefficient β, for the three versions of the model where agents forecast the price directly (**LtF** and **OPT_F**) and indirectly (**IND_F**). In the **LtF** model specification, the median agent learns to chase the price trend with a forecasting rule approximately equal to

$$\begin{aligned} p^e_{i,t} &= \alpha p_{t-1} + (1-\alpha)p^e_{i,t-1} + \beta(p_{t-1} - p_{t-2}) \\ &\approx 0.87 p_{t-1} + 0.13 p^e_{i,t-1} + 0.75(p_{t-1} - p_{t-2}), \end{aligned} \tag{14}$$

though the median trend coefficient β oscillates to some degree, taking values between 0.6 and 0.9. Nevertheless, the 95% CIs remain wide for both coefficients, indicating significant heterogeneity between particular markets.[16] This outcome replicates the central result by Anufriev et al. (2018), and offers a clear interpretation. The asset market features positive feedback, since price mechanism (5) leads to almost self-fulfilling predictions (with a near unit root coefficient of $20/21 \approx 0.952$). If GA agents experiment with trend-following heuristics while observing a price increase, they predict higher prices in the future. This drives the asset demand, and thus the realized price itself, up. GA agents therefore are further encouraged to extrapolate price trends, which closes the self-reinforcing feedback of investor optimism and pessimism.

LtO GA agents from the **OPT_F** specification, who use an independently optimized price forecast in their trading heuristic, converge to a similar behavior. In particular, the median agent learns forecasting heuristic

$$p^e_{i,t} \approx 0.92 p_{t-1} + 0.08 p^e_{i,t-1} + 0.95(p_{t-1} - p_{t-2}), \tag{15}$$

which, in comparison with the **LtF** heuristic, has a higher trend coefficient β (0.95 instead of 0.75). As we will see later in this section, these GA agents learn to put positive weight on the expected asset return in their trading heuristic. It follows that we observe the same mechanism as in the **LtF** case; however, trading amplifies the feedback and hence results in even stronger trend chasing.

In both model specifications, the trend-chasing behavior is only lightly anchored in GA

[16] Note that the 95% CIs for many heuristic coefficients are highly asymmetric. I therefore refrain from using the standard deviation statistic, and focus solely on the shape of the confidence intervals.

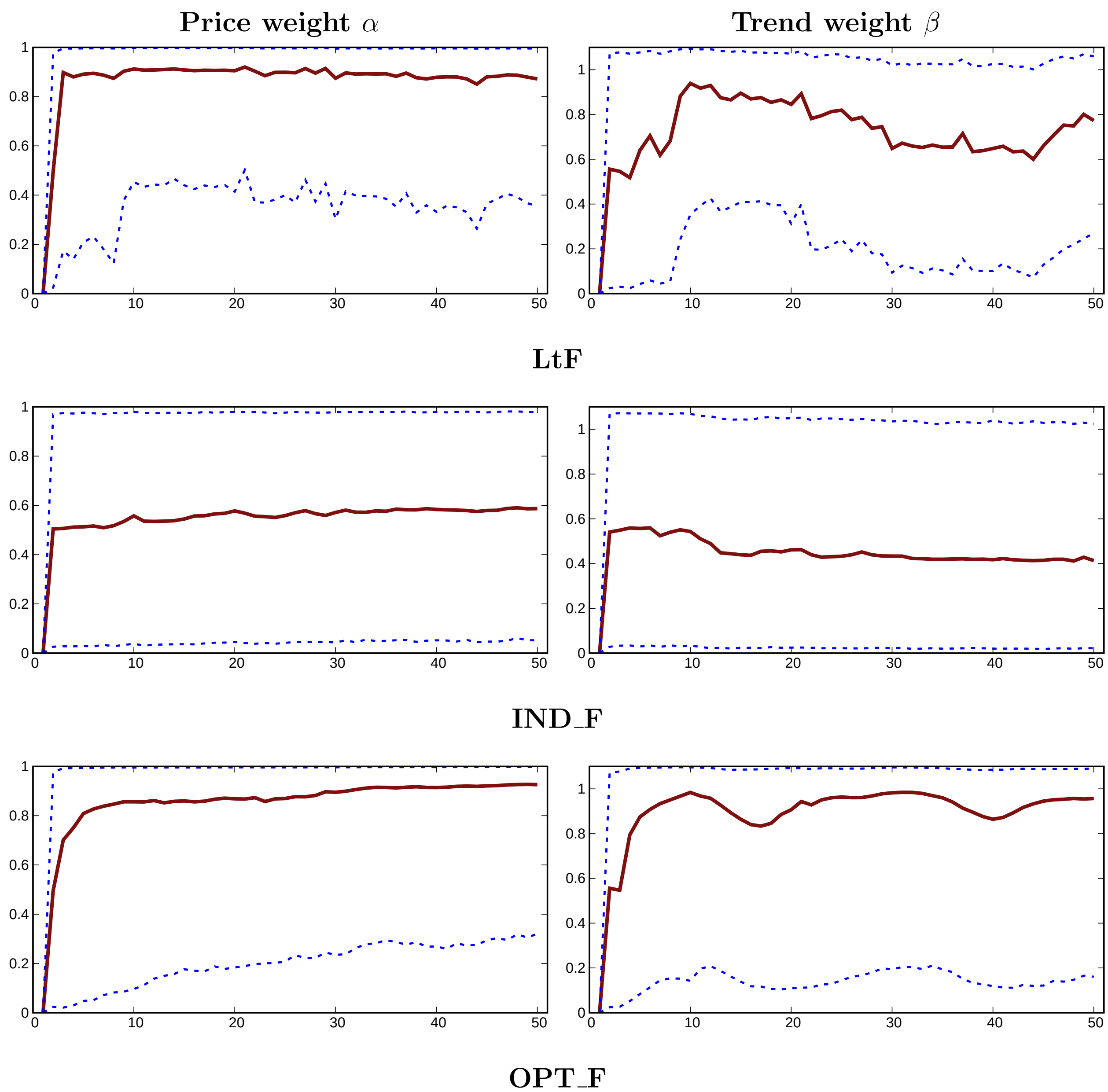


Figure 4: 50-period-ahead MC simulation (1000 markets) for the **LtF**, **IND_F** and **OPT_F** specifications of the GA model: evolution of forecasting heuristics parameters, price weight α (left panels) and trend extrapolation weight β (right panels). The thick red line is the median and the dotted blue lines show the 95% confidence interval for the GA model.

agents' past forecasts (with weight $1-\alpha \approx 0.1$ for the median agent in both specifications). Agents start with a flexible anchor-and-adjustment heuristic, but learn to disregard the anchor, and instead focus on aggressive adjustment strategies.

In contrast to the two previous model specifications, **LtO** GA agents in the **IND_F** model, i.e. agents who use the expected asset return in their trading heuristic, but do not directly optimize the price forecast, follow the price trends to a visibly smaller degree. The median GA agent learns here to use

$$p^e_{i,t} \approx 0.58p_{t-1} + 0.42p^e_{i,t-1} + 0.42(p_{t-1} - p_{t-2}), \tag{16}$$

which is a heuristic that adjusts the anchor relatively conservatively. The simplest interpretation is that GA agents find it difficult to come up with a good price forecasting heuristic in the initial periods (they are unable optimize it directly, after all), therefore they are more likely to keep experimenting with lower β, which reinforces smaller oscillations and thus also the success of milder trend following.

4.3 Trading heuristics

The asset market becomes even more unstable when GA agents try to learn how to directly trade. Figure 5 shows the MC evolution of the parameters of trading heuristics for the three **LtO** model specifications. Unlike for the case of forecasting, the median learning pattern here is not always clear-cut, and large heterogeneity prevails.

The median **NON_F** GA agent converges gradually to trading heuristic

$$\begin{aligned} z_{i,t} &= \chi z_{i,t-1} + \phi \rho_{t-1} \\ &\approx -0.19 z_{i,t-1} + 0.175 \rho_{t-1}. \end{aligned} \tag{17}$$

The anchor element χ in this heuristic comes with a negative sign, so these GA agents learn some degree of mean-reversion behavior. On the other hand, the high coefficient on the past asset return $\phi = 0.175$ (recall that under RE, the coefficient on the expected asset return should be $1/6 \approx 0.167$) means that these GA agents overemphasize the adjustment element of their heuristics and learn to strongly follow market dynamics. In particular, if the asset price rises (falls) sufficiently, the asset return becomes positive (negative). The median **NON_F** agent increases (decreases) her position by more than the RE factor, which feeds back into the price growth (drop). This further inclines GA agents to chase the asset return. On the other hand, the mean-reverting $\chi < 0$ and the lack of a forward-looking component in the trading heuristic imply that, if the market loses some of its momentum, GA agents can reverse their position, and then follow it at a slower pace. This explains why the first bubble in the MC simulations (Figure 3b) bursts, and why the second bubble tends to be smaller than the first one.

The median **IND_F** GA agent converges to a similar heuristic in the form of

$$z_{i,t} \approx -0.16 z_{i,t-1} + 0.16 \rho_{t-1} + 0.15 \rho^e_t. \tag{18}$$

These agents put slightly less emphasis on the previous asset return, but they compensate for it with additional chasing of the expected asset return ρ^e. On the other hand, these agents, given that they are unable to optimize their price heuristic directly, follow the price trends relatively more conservatively than the other GA agents (as seen in equation (16)). Furthermore, the coefficient of the expected asset return is smaller than under RE, with median $\zeta_i \approx 0.15 < 1/6$. This explains why, on the one hand, oscillations in **IND_F** markets do not lose momentum as in the case of the **NON_F** GA model variant, but, on the other hand, are overall the smallest among the trading variants of the model.

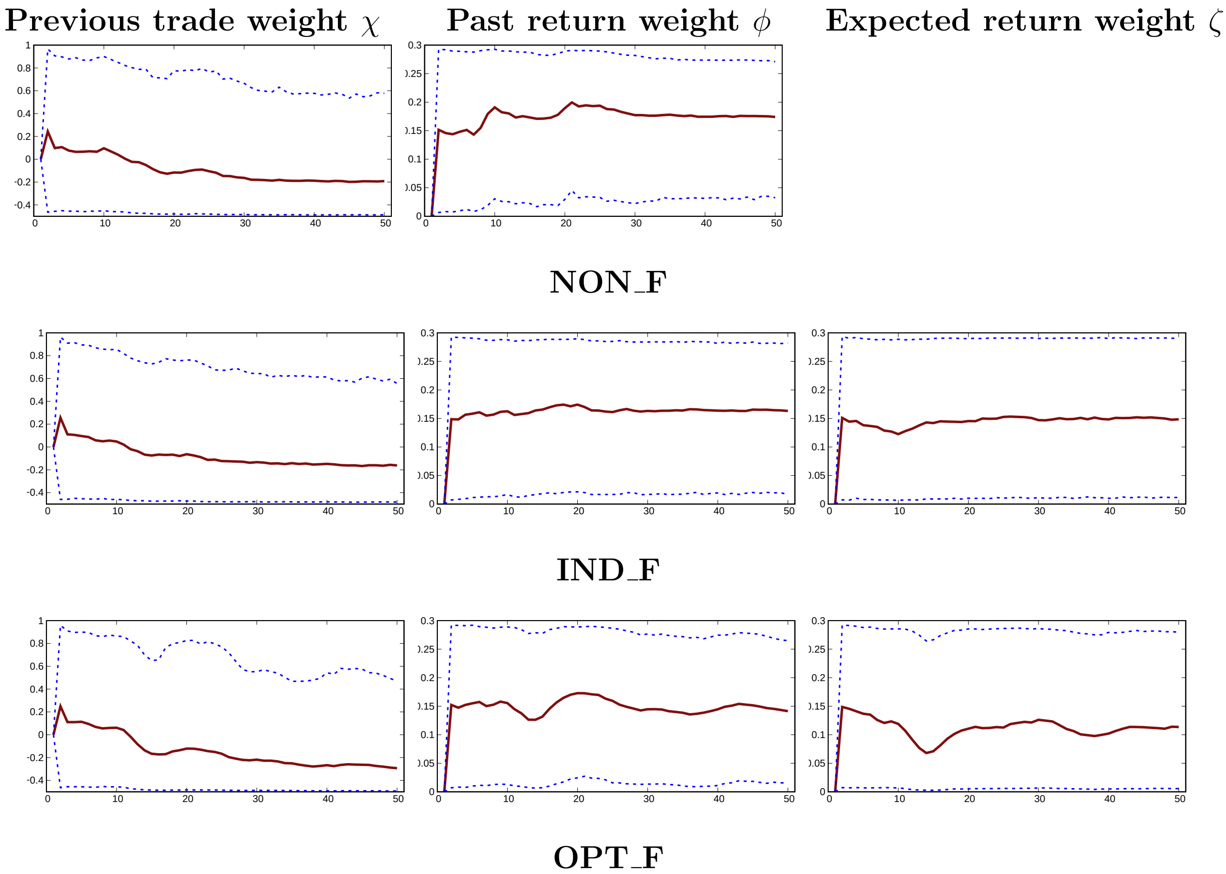


Figure 5: 50-period-ahead MC simulation (1000 markets) for **LtF**, **IND_F** and **OPT_F** specifications of the GA model: evolution of trading heuristics parameters, previous trade weight χ (left panels), past asset return weight ϕ (middle panels) and expected asset return weight ζ (right panels). The thick red line is the median and the dotted blue lines represent the 95% confidence interval for the GA model.

Finally, **OPT_F** GA agents follow a similar pattern to the other two types of trading GA agents. The median agent in this model specification converges to a heuristic in the form of

$$
\begin{aligned}
z_{i,t} &= \chi z_{i,t-1} + \phi \rho_{t-1} + \zeta \rho^e_t \\
&\approx -0.28 z_{i,t-1} + 0.14 \rho_{t-1} + 0.11 \rho^e_t.
\end{aligned} \tag{19}
$$

In comparison with the **NON_F** model specification, the median GA agent chooses an even more negative weight on the anchor (with $\chi \approx -0.28 < -0.19$), and a slightly lower coefficient on the past asset return, which is offset by the additional positive weight on the expected asset return (which **NON_F** GA agents do not use). Even though these agents use smaller ϕ and ζ than **IND_F** GA agents, they also have a much stronger trend-following price heuristic (15). The total effect makes the market in this model specification strikingly unstable.

The two median GA agents in the **IND_F** and the **OPT_F** model specifications learn the ζ coefficient on the expected return, which is similar to the rational solution, but not exactly on the spot. Another deviation from the RE is that both of these agents choose non-zero weights on the previous trade and asset return. *This result is a striking example of how learning and limitations on cognitive abilities can lead to suboptimal behavior in the case of even the most primitive aspects of our decision making,* and how it forms a self-reinforcing feedback system with market dynamics. In fact, Bao et al. (2017) found that only one quarter of their subjects under the Mixed treatment used the optimal trading rule (3). The GA model confirms this to be a robust outcome of learning, instead of an anomaly. In the next section, I provide an interpretation of this result (see Result 3).

4.4 Between and within treatment heterogeneity

The median behavior of each type of GA agents explains general price patterns, which we observe in the simulations of the four variants of the GA model. However, it is important to note that, regardless of the model specification, the 95% CIs of both forecasting and trading parameters remain wide (and so do the CIs of prices). This means that even if GA agents in a particular variant of the model tend to move towards certain behavior in general, large heterogeneity between particular simulations will persist. Some simulated markets can greatly deviate from the dominant type of dynamics, as GA agents, by chance, coordinate on less usual – but still sustainable – feedback between heuristics and price equation.[17] The model heterogeneity explains why the experimental groups generated clear patterns between the treatments, but could still remain highly varied within each treatment. This is particularly clear for **LtO** markets, as seen in the MC price dynamics (Figure 3), and provides further insight into the stylized fact **S2**.

5 A comparison of individual learning between homogeneous model specifications

What drives the differences among the four model variants? Recall that, in each specification, GA agents use different heuristics based on a different parameter space. Therefore, in order to compare the median behavior of GA agents, it is useful to focus on a generalized version of their forecasting and trading heuristics given by

$$z_{i,t} = c_0 y + c_1 z_{i,t-1} + c_2 \rho^e_{i,t-1} + c_3 \Delta p_{t-1} + c_4 p_{t-1}. \tag{20}$$

This simple linear rule, just like its special cases, is an example of an anchor-and-adjustment heuristic. Every GA agent i considers two anchor terms, dividend y and her previous trading position $z_{i,t-1}$. She then adjusts these by extrapolating the last observed price trend $\Delta p_{t-1} = p_{t-1} - p_{t-2}$. At first glance, the previous expected asset return $\rho^e_{i,t-1}$ serves as an anchor, since a positive c_2 relates the agent's present trade to her past beliefs. However, these are beliefs about *market trends*, which is more akin to an adjustment term, and I thus follow this interpretation. Finally, the last observed price level p_{t-1} is an additional anchor term, but in

[17] Appendix B shows the MC results for forecasting and trading coordination of the four variants of the GA model. Figure B.1 shows that, even though some outliers exist, agents remain well coordinated in median simulations of both the **LtF** and all three variants of the **LtO** models. This obviously requires that in the majority of markets, GA agents in particular use similar (albeit not necessarily exactly the same) heuristics.

practice its weight is relatively small, with $c_4 \approx 0$, and hence can be thought of as a residual term (*see* Bao et al., 2017).

The five c_l coefficients of generalized heuristic (20) are directly related to the coefficients of forecasting heuristic (6) of **LtF** GA agents, and the three trading heuristics (7), (9) and (12) of **LtO** GA agents (see Appendix D for derivation). In practice, GA agents in the four model specifications learn to use this rule differently, which explains the discrepancies in the asset price dynamics across the simulations. Table 1 shows the results for the median GA agents from four model specifications.

	Heuristic arguments				
	Dividend y	**Trade** $z_{i,t-1}$	**Exp. return** $\rho^e_{i,t-1}$	**Price trend** Δp_{t-1}	**Price** p_{t-1}
	GA Model: **LtF** specification				
LtF	0.145	0	0.021	0.102	−0.007
	GA Model: **LtO** specifications				
NON_F	0.175	−0.19	0	0.184	−0.088
IND_F	0.247	−0.16	0.063	0.165	−0.012
OPT_F	0.241	−0.28	0.088	0.242	−0.012

Table 1: Coefficients of generalized trading heuristic (20) learned by median GA agents under four different model variants.

There is a clear relationship between the stability of the model specification and the learned behavior, in terms of both the anchor and the adjustment elements of GA agents' heuristics. We can see in Table 1 that the more unstable the specification is (from the most stable **LtF**, through **LtO IND_F** and **NON_F**, to the most unstable **OPT_F**), the higher the two adjustment rates associated with asset return forecast and price trend are. In particular, **LtF** GA agents put a weight of around 0.1 on the previous price trend, while the three types of **LtO** agents use a weight between 1.5 and 2.5 times larger. This is the reason why the trading markets are visibly more unstable, and why they can lead to "super-bubbles".

A second important observation is that GA agents learn more conservative anchoring scheme in the more stable markets. In particular, **LtO** GA agents, in comparison with their more stable **LtF** counterparts, use a negative coefficient on their previous trade, where the order of relative price instability coincides with the absolute value of this coefficient. In other words, the more unstable the environment, the more strongly GA agents learn a mean-reverting behavior. Finally, **LtO** GA agents (in particular, **IND_F** and **OPT_F** agents) learn to put higher weight on dividend y. For the example of the **OPT_F** median agent, the impact of the dividend on her trade is given by $0.241y \approx 0.8$ (where trades are confined into the $z_{i,t} \in [-5, 5]$ interval). This means that, holding all other factors constant, these agents have relatively higher positive bias in their typical demand than **LtF** agents, and are therefore more likely to push the market away from relatively stable price paths.

The theoretical and experimental literature on the LtF environment demonstrates that people use simple anchor-and-adjustment forecasting heuristics, and that feedback exists between the market environment and the type of heuristics that are learned by agents. For the particular example of positive feedback economies (such as asset markets), people learn to follow price trends, which leads to self-fulfilling price oscillations. My GA model shows that, when agents are asked to directly learn how to trade an asset, the learning dynamics pushes them towards the same type of behavior, despite differences in task and framing. In other words, the LtO environment results in agents who are "as-if" price-trend chasing, just like under the LtF regime. If we furthermore ask agents to do both, these two learning patterns amplify each other, resulting in even more extreme or complicated price dynamics.

Result 2. *The GA model confirms the insights of the Learning-to-Forecast literature, and shows that they can be generalized to trading behavior: in asset markets, positive feedback between agent decisions and prices inclines agents to learn to chase price trends.*

Why does trading lead to less stable prices and more trend chasing, as seen in Table 1? The simplest interpretation is based on the Rational Expectations solution to trading and forecasting problems. As explained in the previous sections, both the trading and forecasting heuristics of GA agents have a special "as-if" RE case. However, these have a significantly different structure. Under the **LtF** model specification, GA agents should disregard the adjustment part of their heuristic (the price trend), and rely solely on the anchor equal to the fundamental price, that is set $\alpha = \beta = 0$ and find $p^e_{i,t} = p^f$. In other words, perfectly rational agents under this scenario are trying to learn a *constant*.

Trading agents are faced with the opposite task. Recall that price equation (4) contains small shocks. As a result, the rational demand is on average equal to zero, but its realized value changes from period to period, as seen in equation (8). This implies that traders have to learn *two* things. First, they need to converge to the fundamental forecast (or whatever the rational forecast is in a more general setting). On the top of that, they need to learn how to react to this forecast. Since their trading heuristic is a linear function of the expected asset return, they need to learn the optimal *slope* of that function (in this model, equal to $\zeta^* = 1/6$).

One can imagine that estimating a slope can be more tricky than estimating a constant when the available sample is neither exogenous nor rich. For the particular case of asset markets, they are feedback systems where the previous market conditions (i.e. the sample available to the agents) depends endogenously on agents' behavior (i.e. on what they have previously learned, see Anufriev et al., 2013b, for a discussion and an example in the setting of Industrial Organization). What complicates the learning even more in the context of asset markets is that their feedback is positive, and so renders biases to be self-fulfilling.

Suppose that in some **LtF** market, agents learned to disregard price trends and converged to the neighborhood of the fundamental forecast. In terms of FOR heuristic (6), they understand that $\alpha = \beta = 0$ is the rational rule, but they still need to learn the value of p^f. If they keep on experimenting with the forecast level, trying to learn the exact rational solution, the average forecast will most likely alternate around and very close to the fundamental. To formalize this experimentation, suppose that every agent i forecasts $p^e_{i,t} = p^f + e_{i,t}$, where $e_{i,t} \sim NID(0, v^2)$ is a normally distributed error with some small variance v^2. Thus, price p_t becomes normally distributed as

$$p_t = p^f + \frac{20}{21}\left(\frac{\sum_{i=1}^{6} p^e_{i,t}}{6} - p^f\right) + \varepsilon_t = p^f + \frac{10}{63}\sum_{i=1}^{6} e_{i,t} + \varepsilon_t. \tag{21}$$

This is a positive feedback system, because when the average forecast overshoots the fundamental, the realized price will be also above the fundamental. However, price p_t remains independent from its previous value p_{t-1} on the merit of the price equation itself. As a result, agents have to actively pursue price trends for prices to become serially correlated; and if they do not pursue such trends, the average observed price remains a consistent estimator of the fundamental p^f.

The **LtO** market is very different. There, agents have to learn to forecast the fundamental price, as well as to figure out the optimal slope in their trading heuristic. Consider a market in which **OPT_F** GA agents experiment with their forecast as **LtF** agents in the previous example, and furthermore still need to learn the exact weight on the expected asset return. Formally, they use $\chi = \phi = 0$, $p^e_{i,t} = p^f + e_{i,t}$ and $\zeta_i = 1/6 + \hat{e}_{i,t}$, where $\hat{e}_{i,t} \sim NID(0, \hat{v}^2)$ is a small random error. Then

$$\begin{aligned} p_t =& p_{t-1} + \frac{20}{21}\sum_{i=1} 6\zeta_i \left(p^f + e_{i,t} + y - Rp_{t-1}\right) + \varepsilon_t \\ =& p^f + \frac{20}{21}\left(p^f + y - Rp_{t-1}\right)\sum_{i=1} 6\hat{e}_{i,t} + \sum_{i=6}^{6} e_{i,t}\hat{e}_{i,t} + \varepsilon_t. \end{aligned} \tag{22}$$

The second term $p^f + y - Rp_{t-1}$ is proportional to the deviation from the fundamental of the price from the previous period $t-1$. Unlike in the **LtF** case, random learning mistakes $\hat{e}_{i,t}$ make the current price p_t dependent on the previous price p_{t-1}. For example, suppose that the initial price is below the fundamental, with $p_1 < p^f$ (as typically happens in experimental sessions), which implies that the term $p^f + y - Rp_1$ is positive. If the average agent overreacts (underreacts) such that $\sum_{i=1}^{6} \hat{e}_{i,t} > 0$ ($\sum_{i=1}^{6} \hat{e}_{i,t} < 0$), then the next realized price p_2 is likely to be higher (lower) than the fundamental, just like in the **LtF** case. In this case, however, the deviation of price p_2 from the fundamental level scales up with the previous deviation of p_1 from the fundamental. Note that this holds even when agents have managed to learn the fundamental price itself (with $e_{i,t} = 0$). In other words, experimentation with ζ causes *the price mechanism itself* to amplify this additional volatility, and makes learning much more difficult. If so, agents have even more incentive to abandon the fundamental solution and try to learn some other behavior in comparison with the **LtF** model specification.

In the **LtF** setting, it is easy to coordinate on price trend chasing due to the positive feedback of the market (Hommes, 2013b). This effect becomes stronger under the **LtO** setting. To see this, consider again the **OPT_F** model specification. Suppose that the typical GA agent observes a positive price trend, and extrapolates it, with $\bar{p}_t^e > p_{t-1}$. Assume that she does not care about the previous asset return (with $\phi = 0$), but is over-reactive with $\zeta > 1/6$. As seen in equations (3) and (4), the price grows faster than under rational trading (with $\bar{\zeta} = 1/6$), and yields an even higher realized asset return ρ_t (2), which in turn legitimizes the GA agents' overreaction.

Overreaction, however, does not need to be the sole cause of positive feedback. **OPT_F** GA agents can compensate for lower weight on the expected asset return ζ with positive weight on the previous asset return, with $\phi > 0$, since the realized asset return will remain positive as long as prices continue to grow sufficiently fast. As we saw in the previous section, these GA agents learn to put high weights on both parameters in their trading heuristic (19), together with a strong trend-chasing weight in their forecasting heuristic (15). And if they use such weights, the price again grows even faster than under optimal trading from the **LtF** specification, which in turn justifies higher asset return extrapolation on the top of strong price trend extrapolation. A similar phenomenon occurs for a persistent negative trend: agents switch to short positions that are more aggressive than under rational trading, causing the market to plummet even faster, thus justifying their aggressive betting on price decline. Note that the same reasoning holds for the two other types of **LtO** GA agents.

Either case of $\zeta > 1/6$ or $\phi + \zeta > 1/6$ brings about amplified or *enhanced positive feedback*, where extrapolations of (i) the price trend and (ii) the asset return (last observed or expected) fuel large price oscillations, which in turn fuel aggressive chasing of price trends and asset returns. Since GA agents tend to start far from the fundamental, they are likely to get locked into this regime, and coordinate on highly oscillatory price paths. This explains why **LtO** model specifications tend to generate more unstable price dynamics. In particular, **OPT_F** GA agents learn both price trend-chasing and overreactive extrapolation of expected and previous asset returns, resulting in a series of "super-bubbles" (see Arthur, 2018).

Result 3. *If investors are asked directly to trade, they face "enhanced" positive feedback. If agents extrapolate positive (negative) price trends, but also put a relatively high weight on the previous or expected asset return, they buy (sell) even more of the asset. This makes the price grow (decline) relatively fast, which justifies the behavior of the agents: price-trend seeking and overreactive trading. As a result, traders can generate higher price oscillations than forecasters, who react only to price trends, but not to the dynamics of the asset return.*

6 A mixed market with forecasters and traders

One advantage of an agent-based model is that it allows researchers to study interesting counterfactual scenarios. In this section, I focus on the following question: what happens in a market where some GA agents play the role of **LtF** forecasters and where their trading positions are consistent with forecasts, while the other **LtO** agents have to learn how to perform both tasks? As discussed previously, **LtF** markets tend to be visibly more stable than **LtO** markets. What amount of traders is necessary to further destabilize the off-equilibrium dynamics of pure forecasters' markets and generate "super-bubbles"?

To study this issue, I focus on markets where $J \in \{0, \dots, 6\}$ agents are **OPT_F LtO** GA traders and the remaining $J-6$ are **LtF** GA forecasters. I refer to this model as **MIX_J**. Given that all the three **LtO** markets tend to qualitatively generate the same behavior, it is sufficient to focus on only one type of trader to get insights into the mixed market. Furthermore, **OPT_F** traders optimize their forecasting heuristic the same way **LtF** GA agents do, so this model choice allows for the direct identification of the effect that bounded rationality in trading has on the market on top of behavioral forecasting.

The **MIX_J** model has almost the same structure as the **OPT_F** one. In every period t, agents observe the realized price from the previous period p_{t-1} and update their forecasting heuristics (6) with one GA update based on the relative performance of these heuristics, in order to generate their next forecast $p^e_{i,t}$ (regardless of the type of agent).

Next, **LtF** agents compute optimal demand schedules according to $z^{LtF}_{i,t} = (p^e_{i,t} + y - Rp_{t-1})/6$ as in equation (3). On the other hand, **OPT_F** traders optimize their trading heuristics (12) with one GA update, based on their relative performance. These agents then sample one trading heuristic and use it to compute their individual demand schedules $z^{OPT}_{i,t}$, just like in the baseline **OPT_F** model.

The price is next determined by

$$p_t = p_{t-1} + \frac{20}{21}\sum_{i=1}^{6} z_{i,t} + \varepsilon_t = p_{t-1} + \frac{20}{21}\sum_{i=1}^{J} z^{OPT}_{i,t} + \frac{20}{21}\sum_{i=J+1}^{6} z^{LtF}_{i,t} + \varepsilon_t.$$

GA agents observe p_t, and the market moves to the next period $t+1$. In the simulations, this process repeats itself for $T = 50$ periods. Note that when $J = 6$ ($J = 0$), the model defaults to the homogeneous **OPT_F** (**LtF**) variant.

Detailed MC results for every **MIX_J** model variant can be found in Appendix C. For the sake of simplicity, we focus on the characteristics of the first bubble. Recall that, as seen in Figure 3d, **OPT_F** simulations tended to generate two large bubbles, and the peak of the first one happens during period $t = 14$ (measured by the first peak of the median price and its upper 95% CI). Therefore, the snapshot of the price distribution during period $t = 14$, that is the median price p_{14} and its 95% CI bounds, serves as a natural measure of relative instability. Figure 6 reports these statistics for all possible values of J.

The clear pattern is that, as J goes up, that is as the number of traders rises, the median price and the upper 95% CI bound both increase. On the other hand, the lower bound of the 95% CI initially goes up as well, but it starts to fall for $J = 5$ and $J = 6$. This implies that the more traders relative to forecasters there are, the higher the possible asset bubbles are. However, once traders start to dominate the set of GA agents, markets become more heterogeneous. Typical markets significantly oscillate; however, large outliers (both "super-bubbles" and surprisingly stable or bearish prices) also become more likely.

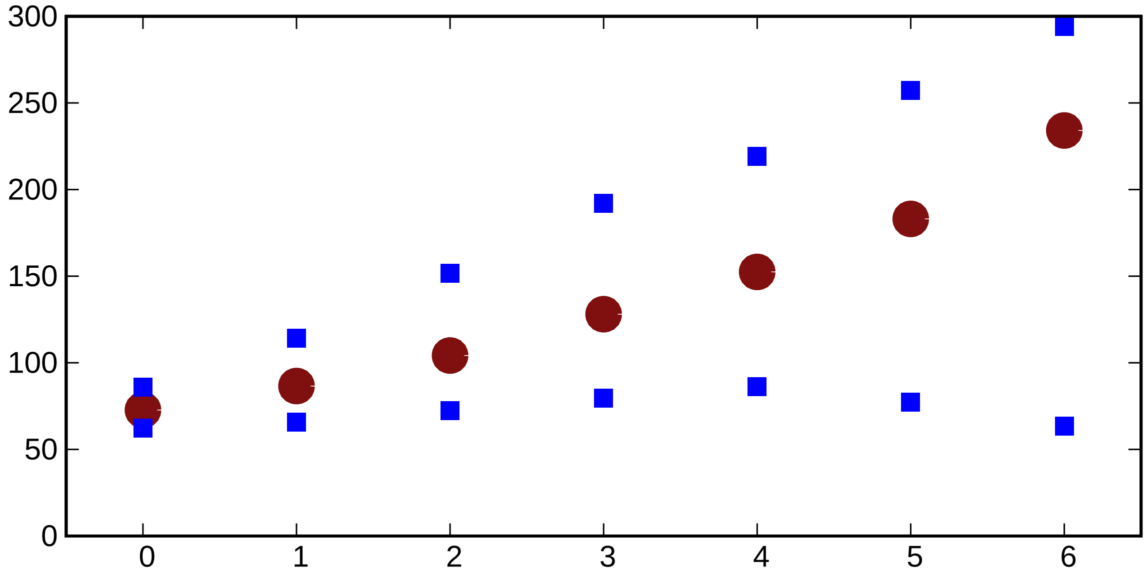


Figure 6: **MIX_J** model for $J \in \{0, \ldots, 6\}$ **LtF** and $6 - J$ **LtO OPT_F** GA agents. Median price p_{14} (red circles) and upper and lower bounds of its 95% Confidence Intervals (blue squares) in period $t = 14$ as a function of J.

	Heuristic arguments				
	Dividend y	**Trade** $z_{i,t-1}$	**Exp. return** $\rho^e_{i,t-1}$	**Price trend** Δp_{t-1}	**Price** p_{t-1}
LtF (J=0)	0.152	0	0.0147	0.129	−0.008
J=1	0.142	0	0.0245	0.128	−0.007
J=2	0.145	0	0.022	0.127	−0.007
J=3	0.147	0	0.0198	0.127	−0.007
J=4	0.149	0	0.0127	0.136	−0.007
J=5	0.16	−0.067	0.01	0.153	−0.008
LtO OPT_F (J=6)	0.184	−0.137	0.01	0.183	−0.009

Table 2: **MIX_J** model for $J \in \{0, \ldots, 6\}$ **LtF** and $6 - J$ **LtO OPT_F** GA agents. Coefficients of the generalized trading heuristic (20) learned by median GA agent.

This corresponds well with the learned behavior. Appendix C provides snapshots of the learned median coefficients of the forecasting and trading heuristics and their 95% CI during period $t = 14$. One can easily compute generalized heuristic (20), which median GA agent exhibited during period $t = 14$ in each of the **MIX_J** models. These results are reported in Table 2.

Consider first the **MIX_J** model variants with fewer than $J = 4$ traders. The weights on the previous trade and price trend are similar here, and equal 0 and $\lesssim 0.13$ respectively. However, for a positive J, the weight on the previous expected asset return is visibly higher, which causes less stable market dynamics and a higher median bubble during period $t = 14$.

The median heuristic undergoes a structural change once the number of **LtO OPT_F** GA traders exceeds the threshold of $J \geqslant 4$. The weight on the previous expected asset return drops by a factor of half, whereas the median agent starts to put more emphasis on the observed price trend Δp_{t-1}, and eventually adopts a mean-reverting heuristic with the previous trade weight dropping from $\chi = 0$ to negative $\chi \approx -0.1$.

This result can be interpreted as a *tipping point*. In Section 5 we saw that **LtO** GA agents, who have to learn how to trade conditional on their forecasts, eventually trade more aggressively than under rational trading, to which **LtF** agents are constrained. It is obvious that these traders can impose their overconfidence on a market only if they are abundant enough, which is approximately half of the market in the GA model. On the other hand, traders can coordinate on more heterogeneous price paths, hence if they dominate over forecasters, some of the markets will become relatively stable or bearish, even if the typical dynamics are by and large bullish. *The existence of the tipping point in the learned behavior demonstrates that our simple linear one-period ahead asset market becomes a complex system under individual learning.*

Result 4. *In a mixed market with both GA traders and forecasters, there exists a tipping point where, if the number of traders is sufficiently high, price oscillations become more heterogeneous and have a higher possible amplitude.*

7 Conclusions

Taming financial instability remains one of the most important policy tasks after the Great Recession of 2008. Literature on behavioral finance has demonstrated that the Efficient Market Hypothesis does not have to necessarily hold, and instead, asset markets can generate off-equilibrium cycles of booms and busts. This is confirmed by experimental evidence, which shows that laboratory subjects often coordinate on non-rational price bubbles. The reason is the *positive feedback character of asset markets*: if agents are optimistic about asset's future profitability, they invest more in that asset, which in turn increases its price. This results in self-reinforcing dynamics between unstable market dynamics and financial agents learning to chase price trends.

The stylized findings of the experimental literature, however, are still subject to one puzzle: *what exactly causes the off-equilibrium learning dynamics of subjects?* Are financial markets unstable because agents fail to learn to forecast asset prices in a rational fashion, or because they fail to translate their forecasts into optimal trading strategies – or maybe both? The literature on this topic is scarce, but existing studies suggest that, in fact, subjects find both tasks difficult (Bao et al., 2017; Nickerson et al., 2007).

In this paper, I studied a heterogeneous agent model based on Anufriev et al. (2018), in which agents were asked to forecast the price of an asset, trade it, or both, in the setting of the Bao et al. (2017) experimental economy. Agents did not know the rational solution to that market, and instead relied on simple anchor-and-adjustment heuristics in the spirit of Tversky and Kahneman (1974). These general heuristics required specific parametrization, and agents *individually* learned one by updating their rules with Genetic Algorithms. Forecasting agents utilized a general adaptive expectation rule with an additional price trend extrapolation component. Three types of traders were assumed. The first type used a heuristic in which the previous position was adjusted by the last observed asset return. The second and third types added to this the expected asset return; the second type learned only how to use that trading rule, whereas the third type also directly optimized her forecasting heuristic.

The analysis of the model yielded three important contributions. First, the model replicates the stylized results of the Learning-to-Forecast and Learning-to-Trade financial experiments, in particular the experiment by Bao et al. (2017). All types of agents learned to chase price trends, which resulted in significant price oscillations. However, the trading agents could coordinate on more diversified oscillations, including "super-bubbles", in which the price repeatedly shifted between 10% and 350% of the fundamental price (as in the most extreme groups in the experiment by Bao et al. (2017)). Second, the paper offers a unified framework for the Learning-to-Forecast and Learning-to-Optimize strands of the behavioral literature, and demonstrates that the insights of the former are also relevant for the latter. The positive feedback aspect of asset markets pushes both forecasters and traders towards symmetric behavior, where they learn to extrapolate price instability as a response to the phenomenon itself, thus reinforcing it. Third, the paper demonstrates that the trend-chasing biases of forecasters and traders amplify each other. When agents had to perform both tasks, market dynamics became particularly unstable, and agents adopted even higher degrees of overreaction, in comparison to the case of markets with only forecasters or only traders. Simulations suggested that this tipping point occurs when at least half of agents are traders. These results directly contradict the assumptions of the literature based on Muth (1961) by demonstrating that asset price oscillations are a robust outcome of individual learning for realistic financial markets.

References

Anufriev, M. and Hommes, C. (2012). Evolutionary selection of individual expectations and aggregate outcomes in asset pricing experiments. *American Economic Journal: Microeconomics*, 4(4):35–64.

Anufriev, M., Hommes, C., and Makarewicz, T. (2018). Simple forecasting heuristics that make us smart: Evidence from different market experiments. *Journal of European Economic Association*, in press.

Anufriev, M., Hommes, C. H., and Philipse, R. H. (2013a). Evolutionary selection of expectations in positive and negative feedback markets. *Journal of Evolutionary Economics*, 23(3):663–688.

Anufriev, M., Kopányi, D., and Tuinstra, J. (2013b). Learning cycles in Bertrand competition with differentiated commodities and competing learning rules. *Journal of Economic Dynamics & Control*, 37(12):2562–2581.

Arifovic, J. (1995). Genetic algorithms and inflationary economies. *Journal of Monetary Economics*, 36(1):219–243.

Arifovic, J. (1996). The behavior of the exchange rate in the genetic algorithm and experimental economies. *Journal of Political Economy*, 104(3):510–541.

Arifovic, J. and Duffy, J. (2018). Heterogeneous agent modelling: experimental evidence. In *Handbook of Computational Economics IV*. Elsevier Amsterdam.

Arthur, W. B. (2018). Asset pricing under endogenous expectations in an artificial stock market. In *The economy as an evolving complex system II*, pages 31–60. CRC Press.

Bao, T., Chia, W. M., and Zhu, J. (2018). Evolutionary selection of forecasting and quantity decision rules in experimental asset markets. EGC Report No.: 2018/07.

Bao, T., Hommes, C., and Makarewicz, T. (2017). Bubble formation and (in) efficient markets in learning-to-forecast and optimise experiments. *The Economic Journal*, 127(605):F581–F609.

Bao, T., Hommes, C., Sonnemans, J., and Tuinstra, J. (2012). Individual expectations, limited rationality and aggregate outcomes. *Journal of Economic Dynamics & Control*, 36(8):1101 – 1120.

Barberis, N. and Thaler, R. (2003). A survey of behavioral finance. *Handbook of the Economics of Finance*, 1:1053–1128.

Bolt, W., Demertzis, M., Diks, C. G., Hommes, C. H., and van der Leij, M. (2014). Identifying booms and busts in house prices under heterogeneous expectations. Tinbergen Institute Discussion Paper, No. 14-157/II.

Boswijk, H. P., Hommes, C. H., and Manzan, S. (2007). Behavioral heterogeneity in stock prices. *Journal of Economic Dynamics & Control*, 31(6):1938–1970.

Breaban, A. and Noussair, C. N. (2015). Trader characteristics and fundamental value trajectories in an asset market experiment. *Journal of Behavioral and Experimental Finance*, 8:1–17.

Brock, W. A. and Hommes, C. H. (1998). Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics & Control*, 22(8):1235–1274.

Colasante, A., Alfarano, S., Camacho, E., and Gallegati, M. (2018). Long-run expectations in a learning-to-forecast experiment. *Applied Economics Letters*, 25(10):681–687.

Costa-Gomes, M. A. and Weizsäcker, G. (2008). Stated beliefs and play in normal-form games. *The Review of Economic Studies*, 75(3):729–762.

Crawford, V. P., Costa-Gomes, M. A., and Iriberri, N. (2013). Structural models of nonequilibrium strategic thinking: Theory, evidence, and applications. *Journal of Economic Literature*, 51(1):5–62.

Dawid, H. (1996). *Adaptive learning by genetic algorithms: Analytical results and applications to economic models*. Springer-Verlag New York, Inc.

Dieci, R., Schmitt, N., and Westerhoff, F. (2018). Interactions between stock, bond and housing markets. *Journal of Economic Dynamics & Control*.

Dieci, R. and Westerhoff, F. (2010). Heterogeneous speculators, endogenous fluctuations and interacting markets: a model of stock prices and exchange rates. *Journal of Economic Dynamics & Control*, 34(4):743–764.

Dieci, R. and Westerhoff, F. (2012). A simple model of a speculative housing market. *Journal of Evolutionary Economics*, 22(2):303–329.

Diks, C. and Makarewicz, T. (2013). Initial predictions in learning-to-forecast experiment. In Teglio, A., Alfarano, S., Camacho-Cuena, E., and Ginés-Vilar, M., editors, *Managing Market Complexity*, volume 662, pages 223–235. Springer Berlin Heidelberg.

Duffy, J. and Tavits, M. (2008). Beliefs and voting decisions: A test of the pivotal voter model. *American Journal of Political Science*, 52(3):603–618.

Duffy, J. and Ünver, M. U. (2006). Asset price bubbles and crashes with near-zero-intelligence traders. *Economic Theory*, 27(3):537–563.

Dufwenberg, M., Lindqvist, T., and Moore, E. (2005). Bubbles and experience: An experiment. *American Economic Review*, pages 1731–1737.

Frijns, B., Lehnert, T., and Zwinkels, R. (2010). Behavioral heterogeneity in the option market. *Journal of Economic Dynamics & Control*, 34(11):2273 – 2287. Special Issue: 2008 Annual Risk Management Conference held in Singapore from June 30 to July 2, 2008.

Gode, D. K. and Sunder, S. (1993). Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality. *Journal of Political Economy*, 101(1):119–137.

Haupt, R. and Haupt, S. (2004). *Practical Genetic Algorithms*. John Wiley & Sons, Inc., New Jersey, 2nd edition.

Heemeijer, P., Hommes, C., Sonnemans, J., and Tuinstra, J. (2009). Price stability and volatility in markets with positive and negative expectations feedback: An experimental investigation. *Journal of Economic Dynamics & Control*, 33(5):1052–1072.

Hommes, C. (2011). The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic Dynamics & Control*, 35(1):1–24.

Hommes, C. (2013a). *Behavioral rationality and heterogeneous expectations in complex economic systems*. Cambridge University Press.

Hommes, C. (2013b). Reflexivity, expectations feedback and almost self-fulfilling equilibria: economic theory, empirical evidence and laboratory experiments. *Journal of Economic Methodology*, 20(4):406–419.

Hommes, C. and Lux, T. (2013). Individual expectations and aggregate behavior in learning to forecast experiments. *Macroeconomic Dynamics*, 17(2):373–401.

Hommes, C., Sonnemans, J., Tuinstra, J., and van de Velden, H. (2007). Learning in cobweb experiments. *Macroeconomic Dynamics*, 11(Supplement S1):8–33.

Hommes, C., Sonnemans, J., Tuinstra, J., and Velden, H. v. d. (2005). Coordination of expectations in asset pricing experiments. *The Review of Financial Studies*, 18(3):pp. 955–980.

Hommes, C. H. and Wagener, F. O. (2009). Complex evolutionary systems in behavioral finance. In *Handbook of financial markets: Dynamics and evolution*, pages 217–276. Elsevier.

Kirchler, M., Huber, J., and Stöckl, T. (2012). Thar she bursts: Reducing confusion reduces bubbles. *American Economic Review*, 102(2):865–883.

Kouwenberg, R., Zwinkels, R., et al. (2010). Chasing trends in the US housing market. Erasmus University Rotterdam working paper.

LeBaron, B. (2006). Agent-based computational finance. *Handbook of Computational Economics*, 2:1187–1233.

LeBaron, B. (2012). Heterogeneous gain learning and the dynamics of asset prices. *Journal of Economic Behavior & Organization*, 83(3):424–445.

Lei, V., Noussair, C. N., and Plott, C. R. (2001). Nonspeculative bubbles in experimental asset markets: Lack of common knowledge of rationality vs. actual irrationality. *Econometrica*, 69(4):831–859.

Lux, T. (1995). Herd behaviour, bubbles and crashes. *The Economic Journal*, pages 881–896.

Lux, T. (2012). Estimation of an agent-based model of investor sentiment formation in financial markets. *Journal of Economic Dynamics & Control*, 36(8):1284–1302.

Marimon, R., Spear, S. E., and Sunder, S. (1993). Expectationally driven market volatility: an experimental study. *Journal of Economic Theory*, 61(1):74–103.

Muth, J. F. (1961). Rational expectations and the theory of price movements. *Econometrica*, 29(3):pp. 315–335.

Nickerson, M., Lahav, Y., and Noussair, C. N. (2007). Traders' expectations in asset markets: experimental evidence. *American Economic Review*, 97(5):1901–1920.

Northcraft, G. B. and Neale, M. A. (1987). Experts, amateurs, and real estate: An anchoring-and-adjustment perspective on property pricing decisions. *Organizational Behavior and Human Decision Processes*, 39(1):84–97.

Noussair, C. N. and Tucker, S. (2013). Experimental research on asset pricing. *Journal of Economic Surveys*.

Palan, S. (2013). A review of bubbles and crashes in experimental asset markets. *Journal of Economic Surveys*, 27(3):570–588.

Rutström, E. E. and Wilcox, N. T. (2009). Stated beliefs versus inferred beliefs: A methodological inquiry and experimental test. *Games and Economic Behavior*, 67(2):616–632.

Sewell, M. (2007). Behavioural finance. *University of Cambridge*.

Shiller, R. (1981). Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review*, 71(3):421–436.

Smith, V., Suchanek, G., and Williams, A. (1988). Bubbles, crashes, and endogenous expectations in experimental spot asset markets. *Econometrica: Journal of the Econometric Society*, pages 1119–1151.

Tversky, A. and Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science*, 185(4157):1124–1131.

Vriend, N. J. (2000). An illustration of the essential difference between individual and social learning, and its consequences for computational analyses. *Journal of Economic Dynamics & Control*, 24(1):1–19.

Weber, M., Duffy, J., and Schram, A. (2018). An experimental study of bond market pricing. *The Journal of Finance*, 73(4):1857–1892.

Westerhoff, F. H. and Reitz, S. (2003). Nonlinearities and cyclical behavior: the role of chartists and fundamentalists. *Studies in Nonlinear Dynamics & Econometrics*, 7(4).

A GA parametrization

Parameter	Notation	Value
Number of agents	I	6
Number of heuristics of one type per agent	H	20
Number of bits per parameter	L_g	20
Mutation rate	δ_m	0.01
Crossover rate	δ_c	0.9
Forecasting heuristic		
Allowed α, price weight	$[\alpha_L, \alpha_H]$	$[0, 1]$
Allowed β, trend extrapolation coefficient	$[\beta_L, \beta_H]$	$[0, 1.1]$
Lower crossover cutoff point	C_L	20
Higher crossover cutoff point	C_H	-1 (none)
Performance measure	$U(\cdot)$	$\exp(-\text{SE}(\cdot))$
Trading heuristic – all LtO variants		
Allowed χ, trade weight	$[\chi_L, \chi_H]$	$[-0.5, 1]$
Allowed ϕ, asset return weight	$[\phi_L, \phi_H]$	$[0, 0.3]$
Performance measure	$U(\cdot)$	$\exp(-(z_{i,t-1} - z^*_{t-1})^2)$
Trading heuristic – NON_F		
Lower crossover cutoff point	C_L	20
Higher crossover cutoff point	C_H	-1 (none)
Trading heuristic – IND_F and OPT_F		
Allowed ζ, expected asset weight	$[\zeta_L, \zeta_H]$	$[0, 0.3]$
Lower crossover cutoff point	C_L	20
Higher crossover cutoff point	C_H	41

Table A.1: Values of parameters used by GA agents.

B Coordination of GA agents

Figure B.1 shows coordination of GA agents in the each of the four variants of the model. The forecasting coordination is measured as the standard deviation of the six forecasts of GA agents in a given market, in a given period, namely as

$$SD(p_t^e) = \sqrt{\frac{1}{6}\sum_{i=1}^{6}\left(p_{i,t}^e - \bar{p}_t^e\right)^2},$$

where $\bar{p}_t^e = 1/6\sum_{i=1}^{6} p_{i,t}^e$ is the average forecast in that period. Trading coordination is measured by an analogous standard deviation of the six individual trades

$$SD(z_t) = \sqrt{\frac{1}{6}\sum_{i=1}^{6}\left(z_{i,t} - \bar{z}_t\right)^2},$$

where $\bar{z}_t = 1/6\sum_{i=1}^{6} z_{i,t}$ is the average trade in that period.

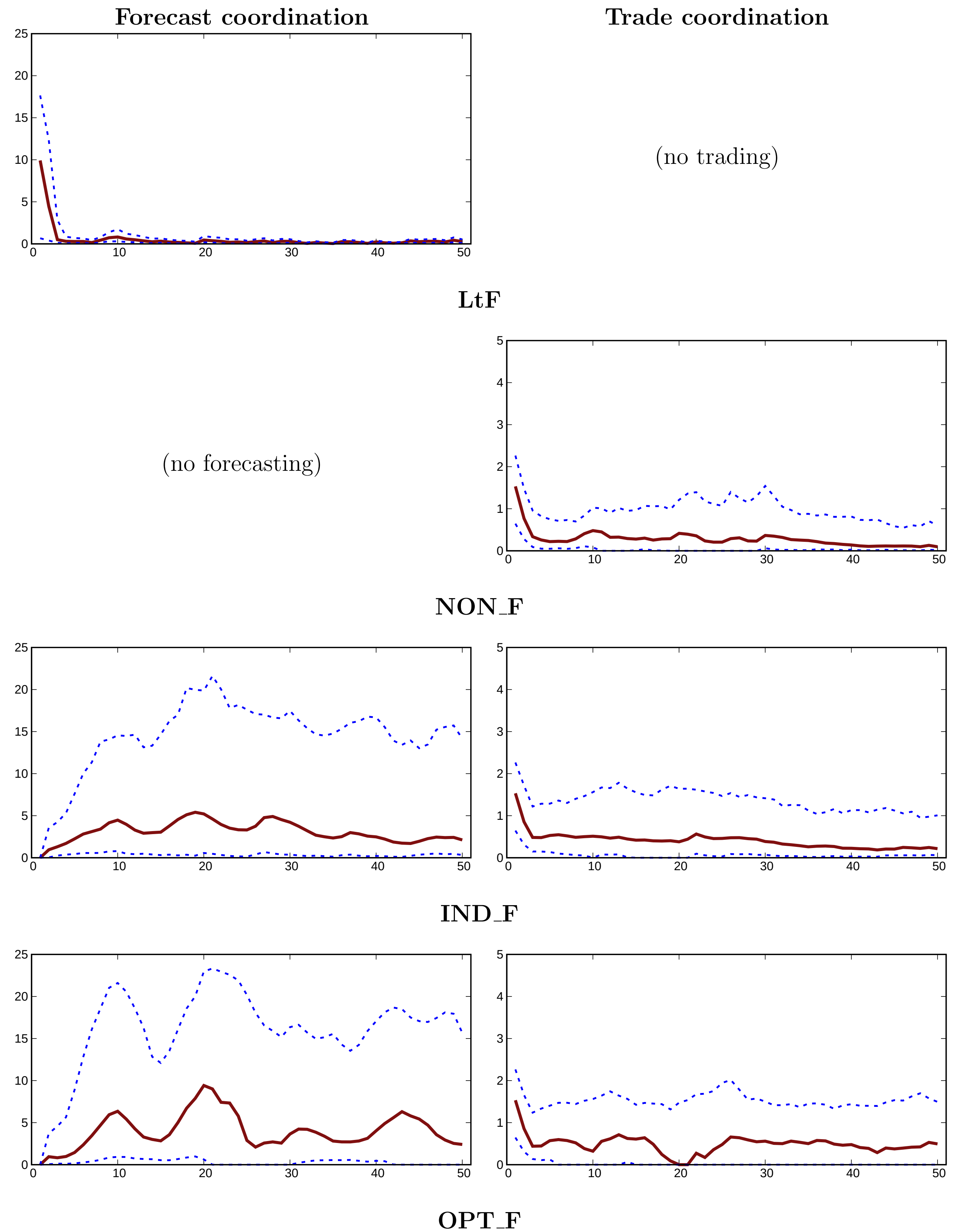
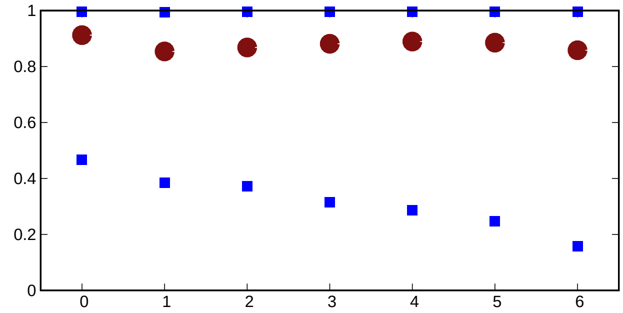
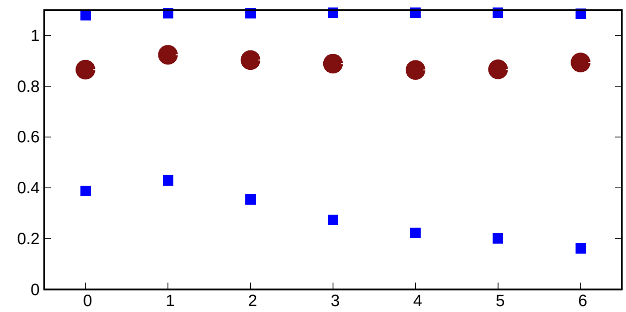


Figure B.1: 50-period-ahead MC simulation (1000 markets) for **LtF**, **NON_F**, **IND_F** and **OPT_F** specifications of the GA model: evolution of forecasting coordination (left panels) and trading coordination (right panels). The thick red line is the median and the dotted blue lines represent the 95% confidence intervals for the GA model.

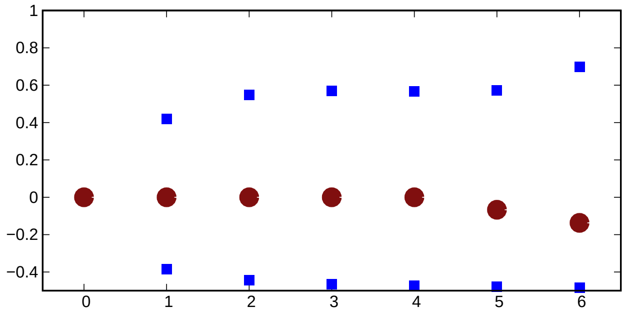
C Monte Carlo results for MIX_J models



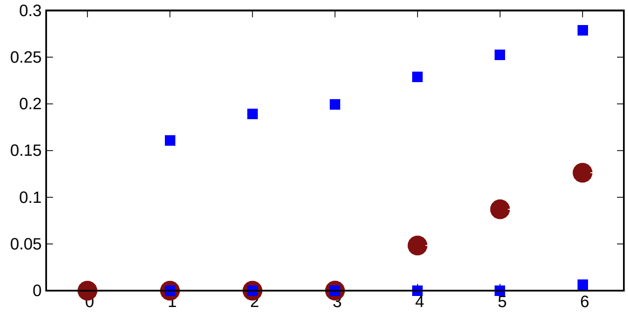
(a) α



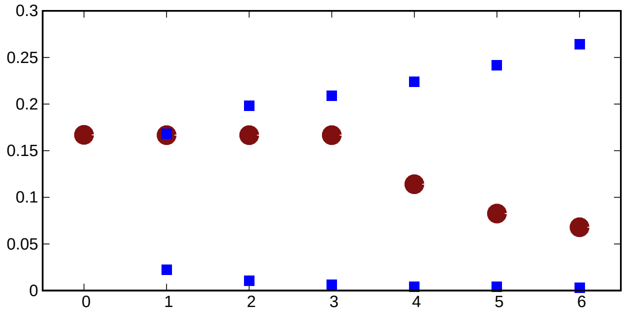
(b) β



(c) χ

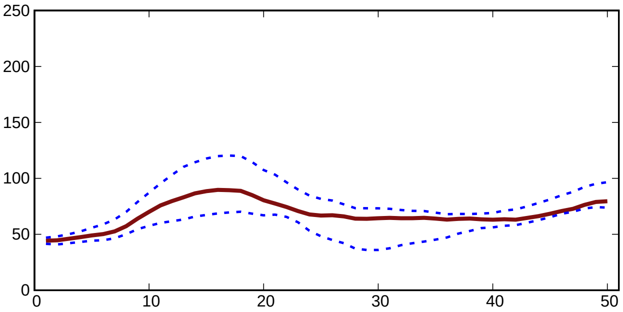


(d) ϕ

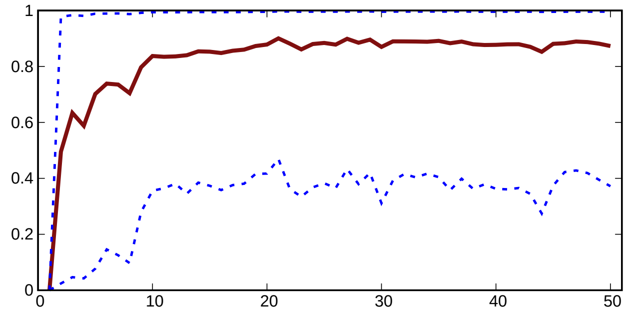


(e) ζ

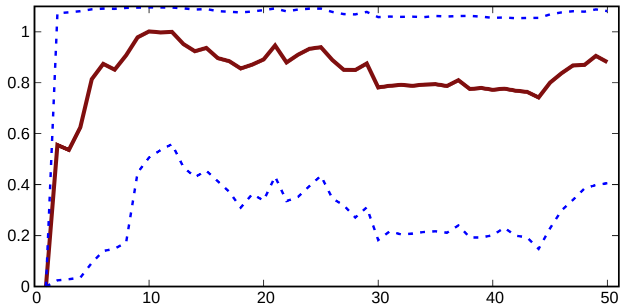
Figure C.1: **MIX_J** model for $J \in \{0, \ldots, 6\}$ **LtF** and $6 - J$ **LtO OPT_F** GA agents. Learned coefficients of forecasting and trading heuristics, price weight α, trend extrapolation weight β, previous trade weight χ, past asset return weight ϕ and expected asset return weight ζ, during period $t = 14$ as a function of J. Median values are denoted by red circles; upper and lower bounds of their 95% CIs by blue squares.



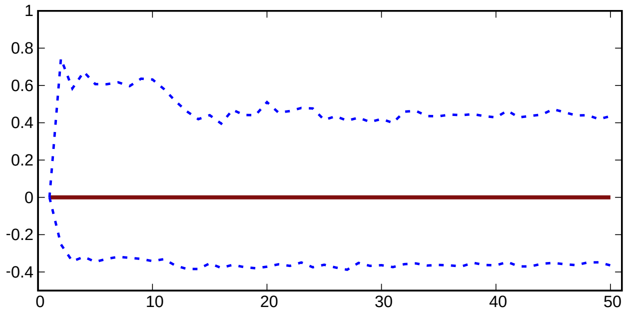
(a) Prices p_t



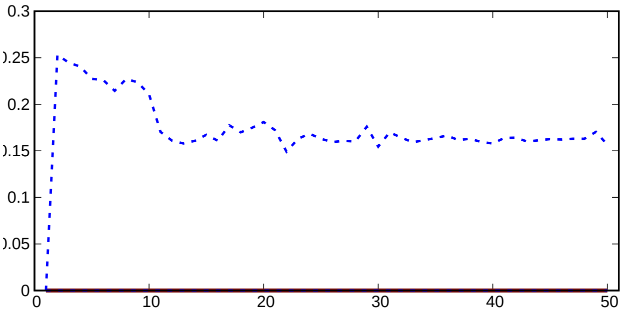
(b) α



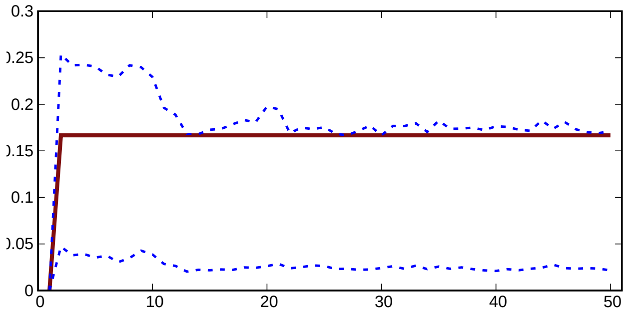
(c) β



(d) χ

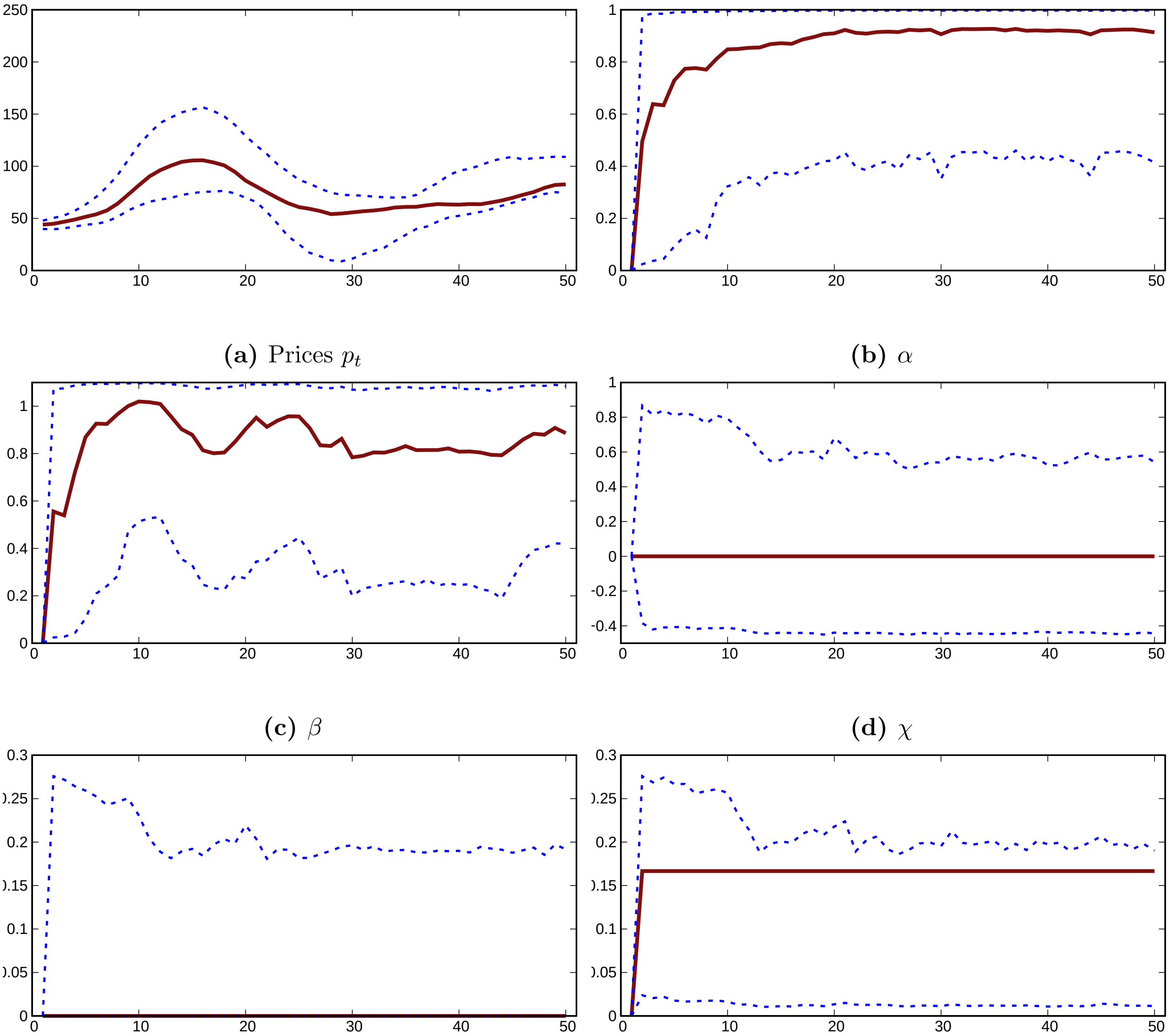


(e) ϕ



(f) ζ

Figure C.2: 50-period-ahead MC simulation (1000 markets) for the **MIX_1** specification of the GA model: evolution of realized prices with forecasting and trading heuristics parameters, price weight α, trend extrapolation weight β (right panels), previous trade weight χ, past asset return weight ϕ and expected asset return weight ζ. The thick red line is the median and the dotted blue lines represent the 95% CI for the GA model.



(a) Prices p_t

(b) α

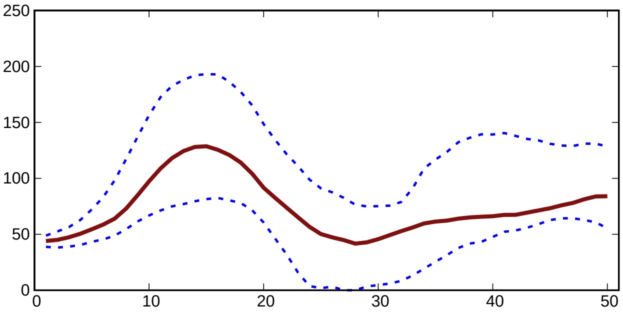
(c) β

(d) χ

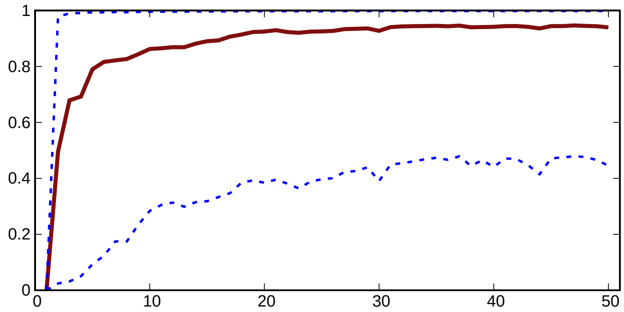
(e) ϕ

(f) ζ

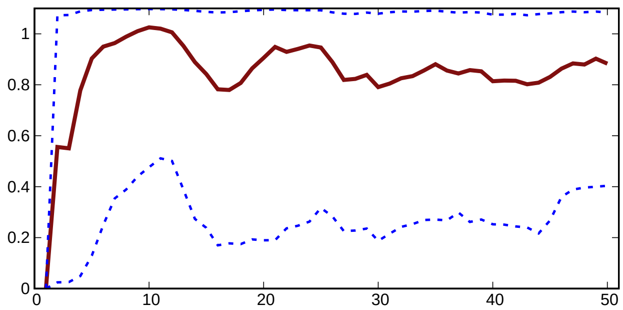
Figure C.3: 50-period-ahead MC simulation (1000 markets) for the **MIX_2** specification of the GA model: evolution of realized prices with forecasting and trading heuristics parameters, price weight α, trend extrapolation weight β (right panels), previous trade weight χ, past asset return weight ϕ and expected asset return weight ζ. The thick red line is the median and the dotted blue lines represent the 95% CI for the GA model.



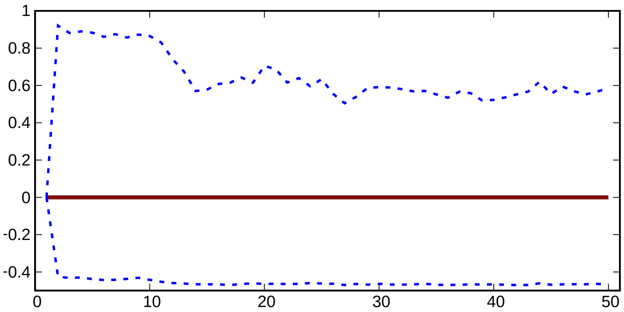
(a) Prices p_t



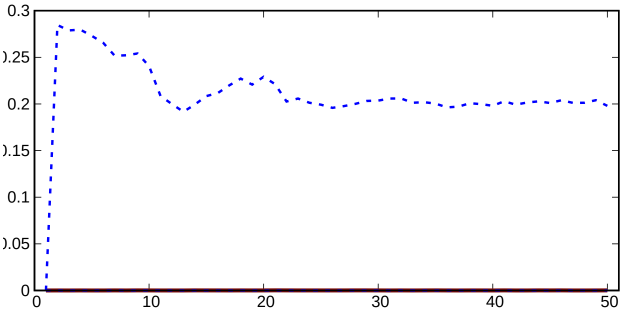
(b) α



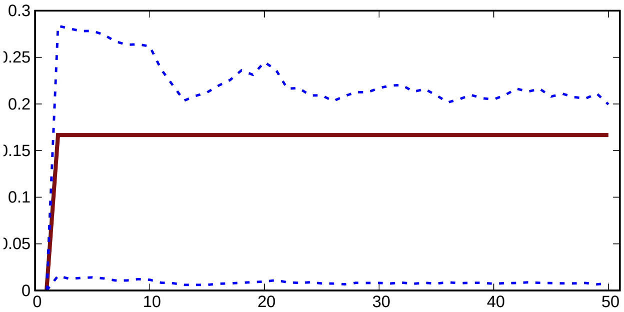
(c) β



(d) χ

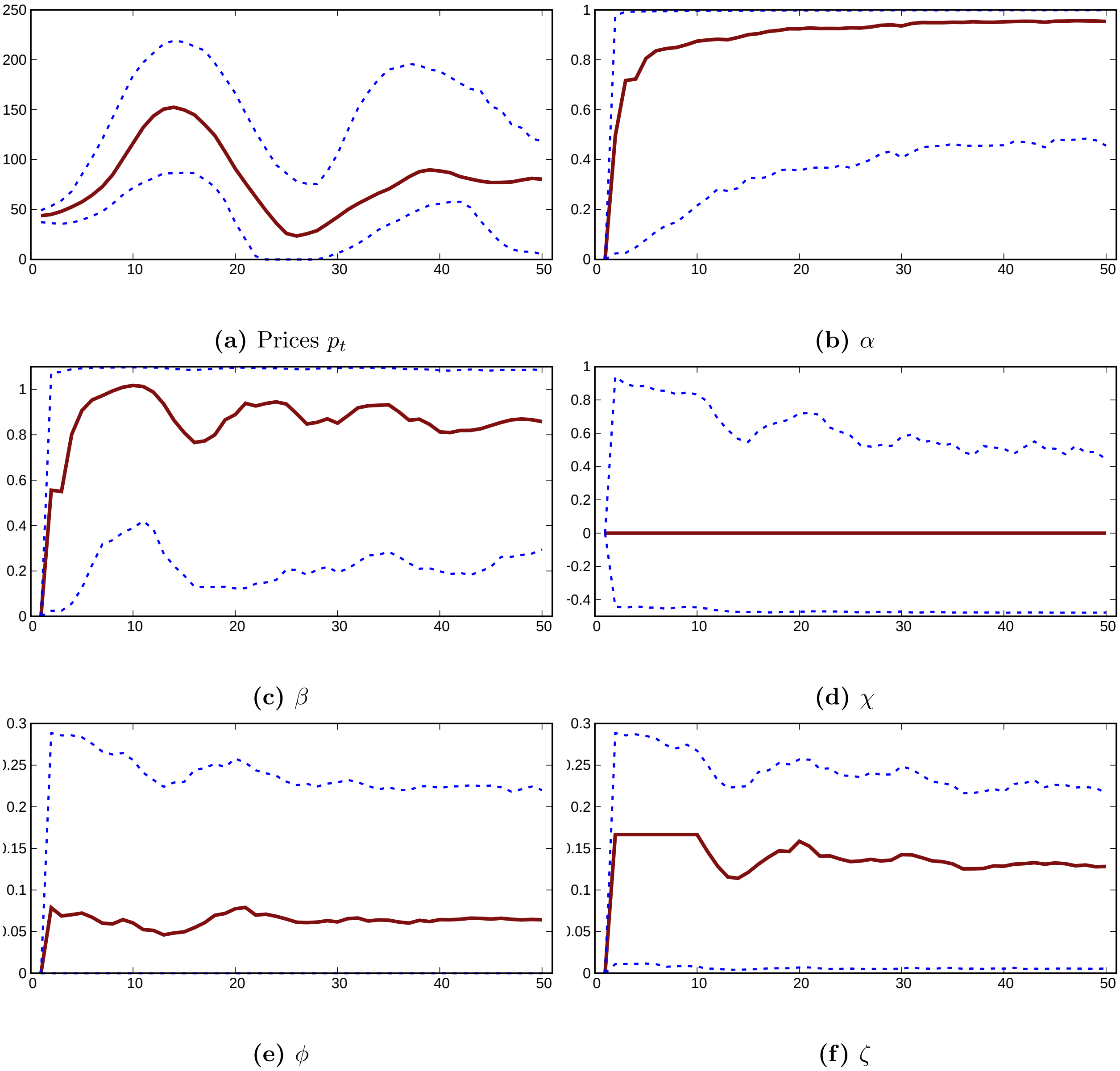


(e) ϕ



(f) ζ

Figure C.4: 50-period-ahead MC simulation (1000 markets) for the **MIX_3** specification of the GA model: evolution of realized prices with forecasting and trading heuristics parameters, price weight α, trend extrapolation weight β (right panels), previous trade weight χ, past asset return weight ϕ and expected asset return weight ζ. The thick red line is the median and the dotted blue lines represent the 95% CI for the GA model.

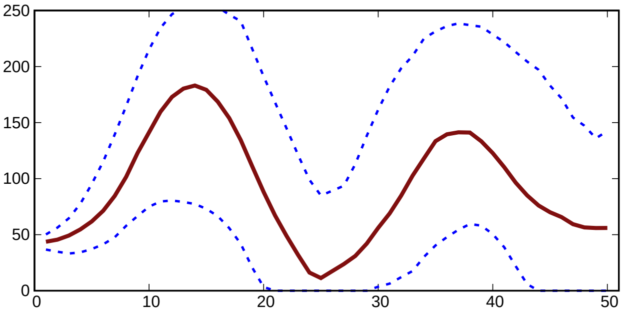


(a) Prices p_t **(b)** α

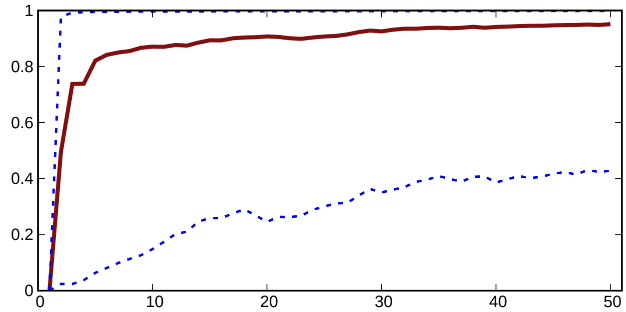
(c) β **(d)** χ

(e) ϕ **(f)** ζ

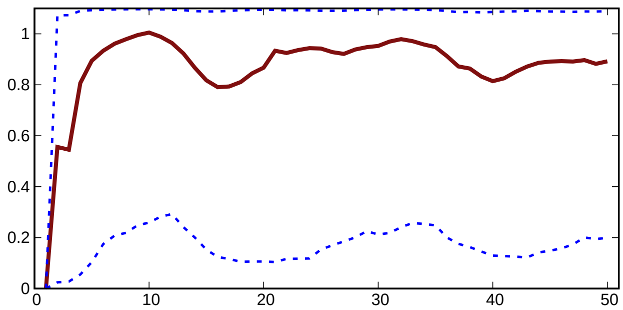
Figure C.5: 50-period-ahead MC simulation (1000 markets) for the **MIX_4** specification of the GA model: evolution of realized prices with forecasting and trading heuristics parameters, price weight α, trend extrapolation weight β (right panels), previous trade weight χ, past asset return weight ϕ and expected asset return weight ζ. The thick red line is the median and the dotted blue lines represent the 95% CI for the GA model.



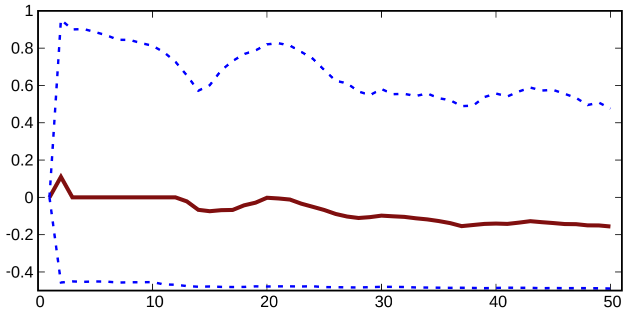
(a) Prices p_t



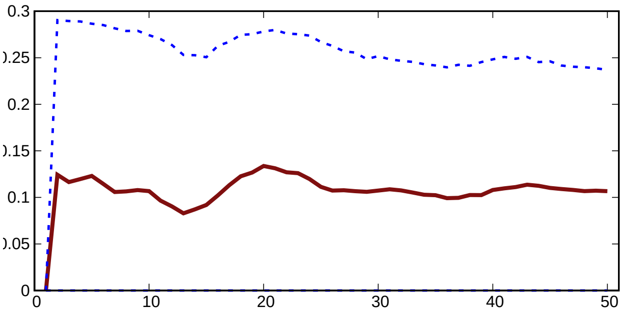
(b) α



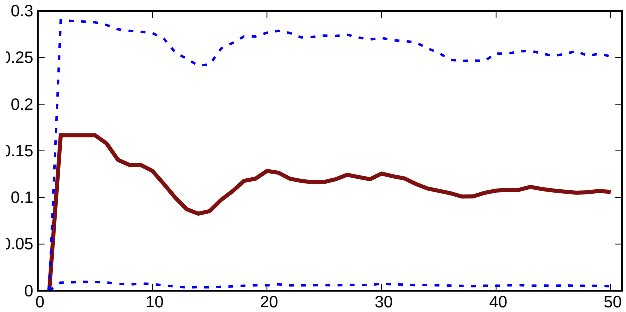
(c) β



(d) χ



(e) ϕ



(f) ζ

Figure C.6: 50-period-ahead MC simulation (1000 markets) for the **MIX_5** specification of the GA model: evolution of realized prices with forecasting and trading heuristics parameters, price weight α, trend extrapolation weight β (right panels), previous trade weight χ, past asset return weight ϕ and expected asset return weight ζ. The thick red line is the median and the dotted blue lines represent the 95% CI for the GA model.

D Generalized anchor-and-adjustment trading/forecasting heuristic

The goal of this appendix is to show that the three trading heuristics of trading GA agents, as well as the trade based on forecasting heuristic (6) of forecasting GA agents, can be represented as a trading rule in the form

$$z_{i,t} = c_0 y + c_1 z_{i,t-1} + c_2 \rho^e_{i,t-1} + c_3 p_{t-1} + c_4 (p_{t-1} - p_{t-2}). \tag{D.1}$$

LtF agents only forecast the price, which is substituted into optimal demand (3). Hence, forecasting heuristic (6) corresponds to the following trading rule:

$$\begin{aligned}
z^F_{i,t} =& (1/6)\left(p^e_{i,t} + y - Rp_{t-1}\right) \\
=& (1/6)\left(\alpha p_{t-1} + (1-\alpha)p^e_{i,t-1} + \beta(p_{t-1} - p_{t-2}) + y - Rp_{t-1}\right) \\
=& \frac{1-\alpha}{6}\left(p^e_{i,t-1} + y - Rp_{t-2}\right) + \frac{1}{6}\left((\alpha + \beta - R)p_{t-1} + (R(1-\alpha) - \beta)p_{t-2} + \alpha y\right) \\
=& \frac{1-\alpha}{6}\rho^e_{i,t-1} + \frac{\alpha}{6} y + \frac{\alpha(1-R)}{6} p_{t-1} + \frac{\beta - (1-\alpha)R}{6}\Delta p_{t-1}.
\end{aligned} \tag{D.2}$$

Note that since **LtF** GA agents always trade optimally conditional on their asset return forecast, an alternative representation of (D.2) can be given by

$$z^F_{i,t} = (1-\alpha) z_{i,t-1} + \frac{\alpha}{6} y + \frac{\alpha(1-R)}{6} p_{t-1} + \frac{\beta - (1-\alpha)R}{6}\Delta p_{t-1}. \tag{D.3}$$

Hence, median forecasting heuristic (14) translates into

$$z^F_{i,t} \approx 0.145y + 0.13 z_{i,t-1} - 0.00725 p_{t-1} + 0.10225 \Delta p_{t-1}. \tag{D.4}$$

NON_F traders adjust their previous trading position in light of the observed asset return, and so heuristic (7) can be rewritten as

$$\begin{aligned}
z^{TN}_{i,t} =& \chi z_{i,t-1} + \phi \rho_{t-1} \\
=& \chi z_{i,t-1} + \phi\left(p_{t-1} + y - Rp_{t-2}\right) \\
=& \phi y + \chi z_{i,t-1} + (1-R)\phi p_{t-1} + R\phi \Delta p_{t-1}.
\end{aligned} \tag{D.5}$$

Thus median heuristic (17) yields

$$z^{TN}_{i,t} \approx 0.175y - 0.19 z_{i,t-1} - 0.0875 p_{t-1} + 0.18375 \Delta p_{t-2}. \tag{D.6}$$

IND_F and **OPT_F** GA traders adjust their previous position in light of both previous and expected returns (though they learn the expected return in a different way). Thus, using $p^e_{i,t} = \alpha p_{t-1} + (1-\alpha)p^e_{i,t-1} + \beta(p_{t-1} - p_{t-2})$, one can show that heuristics (9) and (12) can both be represented as

$$\begin{aligned}
z_{i,t}^{TF} =&\chi z_{i,t-1} + \phi\rho_{t-1} + \zeta\rho_t^e \\
=&\chi z_{i,t-1} + \phi\left(p_{t-1} + y - Rp_{t-2}\right) + \zeta\left(p_{i,t}^e + y - Rp_{t-1}\right) \\
=&\chi z_{i,t-1} + (\phi + \zeta\alpha + \zeta\beta - \zeta R)p_{t-1} - (\phi R + \zeta\beta)p_{t-2} + \zeta(1-\alpha)p_{t-1}^e + (\phi + \zeta)y \\
=&\chi z_{i,t-1} + \zeta(1-\alpha)\left(p_{i,t-1}^e + y - Rp_{t-2}\right) + (\phi + \zeta\alpha + \zeta\beta - \zeta R)p_{t-1} \\
&- (\phi R + \zeta\beta - \zeta(1-\alpha)R)p_{t-2} + (\phi + \zeta - \zeta(1-\alpha))y \\
=&(\phi + \alpha\zeta)y + \chi z_{i,t-1} + \zeta(1-\alpha)\rho_{i,t-1}^e + (1-R)(\phi + \alpha\zeta)p_{t-1} \\
&+ (\phi R + \beta\zeta - (1-\alpha)R\zeta)\Delta p_{t-1}.
\end{aligned} \tag{D.7}$$

It follows that median trading rule (18) of **IND_F** GA agents can be represented as

$$z_{i,t}^{TI} = 0.247y - 0.16z_{i,t-1} + 0.063\rho_{i,t-1}^e - 0.01235p_{t-1} + 0.16485\Delta p_{t-1}, \tag{D.8}$$

while median trading rule (19) of **OPT_F** GA agents can be rewritten as

$$z_{i,t}^{TO} = 0.2412y - 0.28z_{i,t-1} + 0.088\rho_{i,t-1}^e - 0.01206p_{t-1} + 0.24226\Delta p_{t-1}. \tag{D.9}$$

E Initial forecasts and trades

In the experiment by Bao et al. (2017), subjects were given limited information about what to expect in the first period of their session. Under the LtF and Mixed treatments, they were informed that their first forecast should lie in the $[0, 100]$ interval. Under the LtO and Mixed treatment, subjects were instructed that all their trades, including the one in the first period, were constrained by the $[-5, 5]$ interval. Aside from these constraints, subjects were not provided with any additional information, such as a historical sample of prices, forecasts or trades. This implies that their decisions in the first period were – from the perspective of the modeler – purely random.

On the other hand, Diks and Makarewicz (2013) noticed that in the Learning-to-Forecast experiment by Heemeijer et al. (2009) (using the same design as the LtF treatment of Bao et al., 2017), there is actually a structure in subjects' initial forecasts. Around one-third used $p_{i,1}^e = 50$, the midpoint of the initially allowed forecasting area and a natural focal point. Other subjects spread their forecasts more or less uniformly in two uneven intervals, the more popular $[0, 50)$ (forecasts below the focal point) and the less popular $(50, 70]$ (forecasts above the focal point). To model this initial distribution, Diks and Makarewicz (2013) suggested Winged Focal Point distribution (WFP). To be specific, each subject i is assumed to sample her or his initial forecast $p^e_{i,1}$ according to

$$p^e_{i,1} = \begin{cases} \varepsilon^1_i \sim U\left(w_1, w^f\right) & \text{with probability } p_1\,, \\ w^f & \text{with probability } p_2\,, \\ \varepsilon^2_i \sim U\left(w^f, w_2\right) & \text{with probability } 1 - p_1 - p_2\,, \end{cases} \tag{E.1}$$

where w^f is a natural focal point (such as 50 in the LtF treatment), $0 \leqslant w_1 < w^f < w_2 \leqslant 100$ are the limits of "wings" of this distribution, and p_1 and p_2 are the probabilities that subject i will sample her forecast from the left wing, or use the focal point, in the first place. Anufriev et al. (2018) used the estimation technique suggested by Diks and Makarewicz (2013) to calibrate the sampling distribution of the initial forecasts for their three experiments, and argued that this calibration would allow for a Monte Carlo study that would more closely follow the dynamics of the experimental groups.

I used the same modeling solution to initialize my GA model. It was possible to successfully estimate the WFP distribution for the LtF treatment of the experiment by Bao et al. (2017), where

$$p^e_{i,1} = \begin{cases} \varepsilon^1_i \sim U\left(9.375, 50\right) & \text{with probability } 0.25989\,, \\ 50 & \text{with probability } 0.47894\,, \\ \varepsilon^2_i \sim U\left(50, 60.26118\right) & \text{with probability } 0.26117\,. \end{cases} \tag{E.2}$$

On the other hand, subjects under the Mixed treatment did not seem to focus so much on the focal point and the upper wing, and instead the majority of their forecasts seemed to follow a uniform distribution

$$p^e_{i,1} \sim U(41.6016, 50). \tag{E.3}$$

This is an interesting observation that shows that a number of decisions can drastically alter the initial behavior (and possibly – the initial level of sophistication) of laboratory subjects.

On the other hand, visual inspection of initial trades suggests that subjects also sampled these from a WFP distribution, where the focal point was the midpoint of the allowed trades, namely $z_{i,1} = 0$. In this case, the right wing is more pronounced, which is natural, given that it covers positive trades (which is what subjects may have been primed to expect by previous experience or general knowledge). Diks and KS tests (see Diks and Makarewicz, 2013) showed that there was no systematic difference between initial trades in the LtO and Mixed treatments, and that they could be described by WFP distribution in the form

$$z_{i,1} = \begin{cases} \varepsilon^1_i \sim U\left(-3.8826, 0\right) & \text{with probability } 0.49513\,, \\ 0 & \text{with probability } 0.15381\,, \\ \varepsilon^2_i \sim U\left(0, 2.5625\right) & \text{with probability } 0.35106\,. \end{cases} \tag{E.4}$$

In all the simulations for this paper, the respective distributions were used to sample initial decisions. In the **LtF** model variant, GA agents sampled initial forecasts from (E.2). In all three **LtO** model variants, GA agents sampled initial forecasts from (E.3) and initial trades from (E.4). In **Mixed-J** simulations with non-trivial population heterogeneity (i.e. when $J \in \{1, \dots, 5\}$), all trading agents sampled their initial trades from (E.4). For the sake of simplicity, all 6 GA agents (ie., pure forecasters and forecasters-traders) used the same (E.3) distribution to sample initial forecasts. It would have been possible to force the pure forecasters to use the (E.2) distribution instead, but additional sample simulations suggest that this would have had virtually no effect on the model dynamics. The reason is that the mean, standard deviation, and actual structure of (E.2) and (E.3) distributions are comparable.

F Technical details regarding the behavior of GA agents

F.1 Heuristics

Every GA agent $i \in \{1, \dots, 6\}$ at period t has $H = 20$ heuristics indexed by h. The exception is **OPT_F** traders, who have $H = 20$ forecasting and $K = 20$ trading rules. These rules are defined as

$$
\begin{aligned}
\textbf{LtF} \quad p^e_{i,t,h} =& \alpha_{i,t,h} p_{t-1} + (1 - \alpha_{i,t,h}) p^e_{t-1} + \beta_{i,t,h}(p_{t-1} - p_{t-2}) \\
\textbf{NON\_F} \quad z_{i,t,h} =& \chi_{i,t,h} z_{i,t-1} + \phi_{i,t,h} \rho_{t-1} \\
\textbf{IND\_F} \quad z_{i,t,h} =& \chi_{i,t,h} z_{i,t-1} + \phi_{i,t,h} \rho_{t-1} \\
& + \zeta_{i,t,h} \left(\alpha_{i,t,h} p_{t-1} + (1 - \alpha_{i,t,h}) p^e_{t-1} + \beta (p_{t-1} - p_{t-2}) + y - R p_{t-1} \right) \\
\textbf{OPT\_F} \quad p^e_{i,t,h} =& \alpha_{i,t,h} p_{t-1} + (1 - \alpha_{i,t,h}) p^e_{t-1} + \beta_{i,t,h}(p_{t-1} - p_{t-2}) \quad \text{and} \\
z_{i,t,k} =& \chi_{i,t,k} z_{i,t-1} + \phi_{i,t,k} \rho_{t-1} + \zeta_{i,t,k} \left(p^e_{i,t} + y - R p_{t-1} \right)
\end{aligned}
\tag{F.1}
$$

These rules are additionally constrained by $|p^e_{i,t,h} - p_{t-1}| \leqslant 30$ and $z_{i,t,h/k} \in [-5, 5]$. **IND_F** traders remember their implicit forecasts $p^e_{i,t,h} = \alpha_{i,t,h} p_{t-1} + (1 - \alpha_{i,t,h}) p^e_{t-1} + \beta_{i,t,h}(p_{t-1} - p_{t-2})$, so that they can use them during the next period $t + 1$.

Each heuristic is defined as a binary string of 20 bits per heuristic parameter; for example the heuristics of an **IND_F** are encoded as a string of 100 bits, the first 20 bits associated with α, the second with β, and so forth. The exception is again **OPT_F** traders, who each have *two* subsets of heuristics – the first for the 20 forecasting, and the second for the 20 trading heuristics.

F.2 Initial two periods

During the first period $t = 1$, GA agents sampled their decisions from initial exogenous distribution, as presented in Appendix E. In particular,

- in the **LtF** variant of the model, each GA agent i sampled one $p^e_{i,1}$,

- in the **LtO** variant of the model, each GA agent i of type

  – **NON_F** sampled one $z_{i,1}$,

  – **IND_F** sampled one $p^e_{i,1}$ and one $z_{i,1}$,

  – **OPT_F** sampled one $p^e_{i,1}$ and one $z_{i,1}$.

In the **LtF** variant of the model, initial forecasts $p^e_{i,1}$ were substituted directly into price equation (5), while in the **LtO** variant initial trades $z_{i,1}$ were substituted into price equation (4), with $p_0 = 42$ as in the experiment (approximately 64% of the fundamental price).

Once p_1 was generated, GA agents observed it. In the **LtO** variant of the model, agents also learned the first realized asset return $\rho_1 = \rho(p_1, p_0 = 42)$, which they could subsequently use in their trading heuristics $z_{i,2,h}(\cdot)$. On the other hand, **LtF** and **OPT_F** agents observed $\Delta p_1 = p_1 - p_0 = 0$ (assuming no initial price trend).

At the beginning of period $t = 2$, GA agents sampled initial heuristics at random, i.e. every bit in every heuristic of every agent became one or zero with an equal probability of 0.5. Next, each agent i sampled one heuristic (with equal weights of $1/H = 0.05$) to carry out her decision. The exception is **OPT_F** agents, who separately sampled one forecasting heuristic with equal weights and one trading heuristic with equal weights $1/H = 1/K = 0.05$.

F.3 Evaluating the heuristics

Starting with period $t = 3$, each GA agent could evaluate her forecasting and/or trading heuristics from the previous period. In particular, they could focus on the following criteria functions $V(\cdot)$, which measured the hypothetical performance of the heuristics in the previous period:

- **LtF** GA agent i evaluated each forecasting heuristic h with

$$V_F(i,h,t-1) \equiv V_F(p^e_{i,t-1,h}(\alpha_{i,t-1,h},\beta_{i,t-1,h})) = \left(p^e_{i,t-1,h}(\alpha_{i,t-1,h},\beta_{i,t-1,h}) - p_{t-1}\right)^2 . \tag{F.2}$$

- **LtO** GA agents:

  NON_F GA agent i evaluated each trading heuristic h with

$$\begin{aligned} V_{NO}(i,t-1,h) \equiv & V_{NO}(z_{i,t-1,h}(\chi_{i,t-1,h},\phi_{i,t-1,h})) \\ = & \left(z_{i,t-1,h}(\chi_{i,t-1,h},\phi_{i,t-1,h}) - \frac{\rho_{t-1}}{6}\right)^2 , \end{aligned} \tag{F.3}$$

  which shows the squared deviation from the optimal demand at period $t-1$ (3) and is proportional to the realized utility of agent i (1).

IND_F GA agent i evaluated each trading heuristic h with

$$
\begin{aligned}
V_{IND}(i,t-1,h) \equiv & V_{IND}(z_{i,t-1,h}(\chi_{i,t-1,h},\phi_{i,t-1,h},\zeta_{i,t-1,h},\alpha_{i,t-1,h},\beta_{i,t-1,h})) \\
= & \left(z_{i,t-1,h}(\chi_{i,t-1,h},\phi_{i,t-1,h},\zeta_{i,t-1,h},\alpha_{i,t-1,h},\beta_{i,t-1,h}) - \frac{\rho_{t-1}}{6}\right)^2. \qquad \text{(F.4)}
\end{aligned}
$$

OPT_F GA agent i evaluated her forecasting and trading heuristics separately; more specifically, she evaluates each forecasting heuristic h with the squared error measure (similarly to **LtF** agents)

$$
\begin{aligned}
V^F_{OPT}(i,t-1,h) \equiv & V^F_{OPT}(p^e_{i,t-1,h}(\alpha_{i,t-1,h},\beta_{i,t-1,h})) \\
= & \left(p^e_{i,t-1,h}(\alpha_{i,t-1,h},\beta_{i,t-1,h}) - p_{t-1}\right)^2, \qquad \text{(F.5)}
\end{aligned}
$$

and each trading heuristic k with

$$
\begin{aligned}
V^T_{OPT}(i,t-1,k) \equiv & V^T_{OPT}(z_{i,t-1,k}(\chi_{i,t-1,k},\phi_{i,t-1,k},\zeta_{i,t-1,k})) \\
= & \left(z_{i,t-1,k}(\chi_{i,t-1,k},\phi_{i,t-1,k},\zeta_{i,t-1,k}) - \frac{\rho_{t-1}}{6}\right)^2. \qquad \text{(F.6)}
\end{aligned}
$$

Note that measures $V(\cdot)$ are computed treating p_{t-1}, ρ_{t-1}, $p^e_{i,t-2}$ and/or $z_{i,t-2}$ as constants, i.e. agents do not evaluate the effect of the change in their forecasts and trades on past prices. This is a standard assumption of price-taking behavior.

At the beginning of period t, when the information set contains all periods up to and including $t-1$, GA agents use GA operators, and sample *independently* their heuristics, with probability weights defined by the logit transformation

$$
\Pi_{i,t-1,u} = \frac{\exp -V_{i,t-1,u}(\cdot)}{\sum_{\hat{u}\in U} \exp -V_{i,t-1,\hat{u}}(\cdot)} \in [0,1), \qquad \text{(F.7)}
$$

of the relevant criterion function for all heuristics $u \in U \in \{H, K\}$.

BERG Working Paper Series (most recent publications)

108 Jutta **Viinikainen**, Guido **Heineck**, Petri **Böckerman**, Mirka **Hintsanen**, Olli **Raitakari** and Jaakko **Pehkonen**, Born Entrepreneur? Adolescents' Personality Characteristics and Self-Employment in Adulthood, March 2016

109 Stefanie P. **Herber** and Michael **Kalinowski**, Non-take-up of Student Financial Aid: A Microsimulation for Germany, April 2016

110 Silke **Anger** and Daniel D. **Schnitzlein**, Cognitive Skills, Non-Cognitive Skills, and Family Background: Evidence from Sibling Correlations, April 2016

111 Noemi **Schmitt** and Frank **Westerhoff**, Heterogeneity, spontaneous coordination and extreme events within large-scale and small-scale agent-based financial market models, June 2016

112 Benjamin **Lojak**, Sentiment-Driven Investment, Non-Linear Corporate Debt Dynamics and Co-Existing Business Cycle Regimes, July 2016

113 Julio **González-Díaz**, Florian **Herold** and Diego **Domínguez**, Strategic Sequential Voting, July 2016

114 Stefanie Yvonne **Schmitt**, Rational Allocation of Attention in Decision-Making, July 2016

115 Florian **Herold** and Christoph **Kuzmics**, The evolution of taking roles, September 2016.

116 Lisa **Planer-Friedrich** and Marco **Sahm**, Why Firms Should Care for All Consumers, September 2016.

117 Christoph **March** and Marco **Sahm**, Asymmetric Discouragement in Asymmetric Contests, September 2016.

118 Marco **Sahm**, Advance-Purchase Financing of Projects with Few Buyers, October 2016.

119 Noemi **Schmitt** and Frank **Westerhoff**, On the bimodality of the distribution of the S&P 500's distortion: empirical evidence and theoretical explanations, January 2017

120 Marco **Sahm**, Risk Aversion and Prudence in Contests, March 2017

121 Marco **Sahm**, Are Sequential Round-Robin Tournaments Discriminatory?, March 2017

122 Noemi **Schmitt**, Jan **Tuinstra** and Frank **Westerhoff**, Stability and welfare effects of profit taxes within an evolutionary market interaction model, May 2017

123 Johanna Sophie **Quis** and Simon **Reif**, Health Effects of Instruction Intensity – Evidence from a Natural Experiment in German High-Schools, May 2017

124 Lisa **Planer-Friedrich** and Marco **Sahm**, Strategic Corporate Social Responsibility, May 2017

125 Peter **Flaschel**, Matthieu **Charpe**, Giorgos **Galanis**, Christian R. **Proaño** and Roberto **Veneziani**, Macroeconomic and Stock Market Interactions with Endogenous Aggregate Sentiment Dynamics, May 2017

126 Christian **Menden** and Christian R. **Proaño**, Dissecting the Financial Cycle with Dynamic Factor Models, May 2017

127 Christoph **March** and Marco **Sahm**, Contests as Selection Mechanisms: The Impact of Risk Aversion, July 2017

128 Ivonne **Blaurock**, Noemi **Schmitt** and Frank **Westerhoff**, Market entry waves and volatility outbursts in stock markets, August 2017

129 Christoph **Laica**, Arne **Lauber** and Marco **Sahm**, Sequential Round-Robin Tournaments with Multiple Prizes, September 2017

130 Joep **Lustenhouwer** and Kostas **Mavromatis**, Fiscal Consolidations and Finite Planning Horizons, December 2017

131 Cars **Hommes** and Joep **Lustenhouwer**, Managing Unanchored, Heterogeneous Expectations and Liquidity Traps, December 2017

132 Cars **Hommes**, Joep **Lustenhouwer** and Kostas **Mavromatis**, Fiscal Consolidations and Heterogeneous Expectations, December 2017

133 Roberto **Dieci**, Noemi **Schmitt** and Frank **Westerhoff**, Interactions between stock, bond and housing markets, January 2018

134 Noemi **Schmitt**, Heterogeneous expectations and asset price dynamics, January 2018

135 Carolin **Martin** and Frank **Westerhoff**, Regulating speculative housing markets via public housing construction programs: Insights from a heterogeneous agent model, May 2018

136 Roberto **Dieci**, Noemi **Schmitt** and Frank **Westerhoff**, Steady states, stability and bifurcations in multi-asset market models, July 2018

137 Steffen **Ahrens**, Joep **Lustenhouwer** and Michele **Tettamanzi**, The Stabilizing Role of Forward Guidance: A Macro Experiment, September 2018

138 Joep **Lustenhouwer**, Fiscal Stimulus in an Expectation Driven Liquidity Trap, September 2018

139 Tim **Hagenhoff**, An aggregate welfare optimizing interest rate rule under heterogeneous expectations, October 2018

140 Johanna Sophie **Quis**, Anika **Bela** and Guido **Heineck**, Preschoolers' self-regulation, skill differentials, and early educational outcomes, December 2018

141 Tomasz **Makarewicz,** Traders, forecasters and financial instability: A model of individual learning of anchor-and-adjustment heuristics, January 2019